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# Joint tasks and cache partitioning for real-time systems

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# 1 Introduction

A system is a combination of hardware and software intended to interact with the environment. The system detects events using a set of sensors, and acts on the environment through a set of actuators. Some systems must carry out actions in limited time, in order to timely react to the detection of specific events, or perform an action periodically. These systems are known as *real-time* systems.

There are two types of real-time constraints: *soft* real-time constraints and *hard* real-time constraints. A system with soft real-time constraints is authorized to miss its time constraint occasionally without catastrophic consequences. For example, during the decoding of a video, the decoding of a frame should be finished before the frame is displayed. If the decoding is completed too late, the frame is no longer useful and is discarded. If it only happens occasionally, reading is undisturbed. Instead, hard real-time constraints cannot be missed without serious consequences. For example, hard real-time systems are used in avionics and aerospace, areas where missing timing constraints can be dangerous for people.

Validating real-time constraints opens up many research issues. For real-time systems running on single-core processors, the steps are basically: *(i)* to obtain an overestimation of the worst case execution time of all concurrent activities (tasks) to be executed, *(ii)* scheduling tasks over time, *(iii)* checking that all executions allowed by the scheduling policy meet the system timing constraints.

Many studies have been conducted in these areas for single core architectures. Although some issues remain open, most efforts are now focused on real-time systems running on *multi-core processors*.

One of the issues raised by these architectures is the sharing of hardware resources between cores. For example, it is common for these architectures to possess one or more levels of caches shared between cores. Two concurrent programs that run on different cores use the shared cache(s) and can cause cache misses, that would not be present in a execution on single core architecture with private caches. One solution that we will detail further is *cache partitioning*.

Task scheduling on multi-core architecture, in addition to distributing the tasks in time, have to allocate the tasks to the cores. One solution, called *partitioned scheduling* assign a core to each task and forbid task migrations.

So far, most cache partitioning methods have been proposed for single core architectures. Furthermore, most existing task partitioning algorithms

ignore cache effects. To the best of our knowledge, only one research study [13] jointly considers cache partitioning and task partitioning. In [13], an algorithm, called IA3 jointly partitions tasks and caches for non-preemptive scheduling. However, it does not take advantage of all properties of tasks.

In this work we present PDAA, a joint task and cache partitioning algorithm for system scheduled using non-preemptive Earliest Deadline First (NP-EDF). On the one hand, PDAA takes benefit of relationships between task periods for partitioning tasks among cores. On the other hand, tasks cache usage is accounted for in order to partition the shared cache among cores. Experimental results show that PDAA outperforms IA3 in terms of percentage of schedulable task sets.

The remainder of the report is organized as follows. Background material is given in section 2. Notations and problem formulation are given in section 3. Our algorithm, named PDAA, is described in section 4. Experimental results are given in section 5 and show that PDAA outperforms IA3 in terms of task schedulability. A summary of the contributions and directions for future work are finally given in section 6.

## 2 Context and state-of-the-art

### 2.1 Real time systems on single core architectures

In hard real-time systems, all activities have to meet timing constraints, typically termination deadlines. Throughout this document, we will call "task" any process, thread, or another portion of code which is subject to timing constraints.

To guarantee timing constraints it is necessary to know the worst case execution time of every task. We will use the acronym WCET (Worst Case Execution Time) in the rest of the document. When the target architecture is a single core architecture, the WCET of a task depends only on the code of the task and on the architecture, and not on the other tasks running concurrently. Several characteristics of the architecture can influence the WCET. We will focus in this document on the effect of instruction caches.

Once the WCET of each task is known, we must determine when tasks will be executed (task scheduling). Many real-time scheduling strategies exist, depending on whether preemptions are allowed or not, whether or not the schedule is determined off-line, priority assignment, etc. Under a given scheduling policy, verification methods called *schedulability analysis methods* can then be applied to determine if timing constraints are met or not.

### 2.1.1 WCET estimation

The WCET of a task on a given processor is the worst case execution time this task can take to execute in isolation from the other tasks. In general, to keep the complexity of WCET estimation reasonable, an overestimation of the actual WCET of a task is computed. It is then necessary that this overestimated WCET is as close as possible to the actual WCET (*accuracy*), but necessarily greater than or to equal the actual WCET (*safety*). Excessive overestimations may result in a negative schedulability verdict by the schedulability analysis method, forcing the system designer to use a faster, and therefore more expensive, processor.

There are two general approaches to estimate WCETs: *static* and *dynamic* methods. The dynamic methods, also termed *measurement-based*, measure the execution time of every task when executed on the processor (or using a cycle-accurate simulator). In order to be safe, such methods need to identify the input dataset that triggers the execution of the longest path of the program.

In the simplest cases, knowledge of the application code by the application developer can give the worst data set. It is also possible to test all datasets if the application is simple enough to enumerate all input parameter configurations. However, in the general case, the number of different datasets, or even the number of different paths in a tasks is too high for such methods to be practical. Although there are some research works to decrease the complexity of dataset/path enumeration, static methods are preferred.

Static WCET estimation methods, in contrast to dynamic methods, do not execute the tasks' code. Static methods generate a representation of the task (e.g. control flow graph), compute at every program point the worst-case state of hardware, and then use it to compute the WCET. For the computation of worst-case state of hardware, there are many architecture-specific point that can be taking account to improve the tightness of the estimation. For example: the caches, out-of-order execution, pipelining, etc. In this article we will focus on the caches.

**Caches** The difference in speed of accesses to the main memory and processor registers of processors motivate the use of a memory hierarchy made of *caches*. Processors now have several levels of caches : the level 1 (L1) cache is the smaller and closer to the processor, and thus faster; the level 2 and level 3 caches have a higher latency but are larger; the longer the access latency the larger the cache capacity. There are several cache management

policies to control the transfer of information between the different cache levels. A widespread policy is the *non inclusive* policy. When using that policy, when the processor makes a memory access, it first scans the L1 cache; if the data is not present (cache miss), the processor then makes a request to the level 2 cache. This continues until it reaches the main memory. When the requested information is found, it is copied into all traversed cache levels.

In the WCET calculation method evoked above, the cost in time assigned to each memory access is the maximum (usually the time costing a cache miss). However, it is possible to improve the accuracy of WCET calculated taking into account the processor caches .

Consider a processor with one level of cache. [6] Statics analyses are able to determine the result of a cache hit for each reference (CHMC). Thus each memory access may always result in a cache hit, always result in a default cache, result in a cache miss for the first time and then in cache hit, or result in an indeterminate access. Thus it is possible to determine more precisely the cost of each memory access, which improves the accuracy of the WCET.

This analysis was extended [9] to accommodate multiple levels of caches, and several replacement policies. In general the analysis allows determining whether a data will be requested from a cache level  $i$ . For that the level of CHMC  $I - 1$  is used. For example, a reference marked "always cache hit" at  $i - 1$  will never demand at  $i$ .

### 2.1.2 Task scheduling

Task scheduling [5] consists in deciding in which order tasks have to be executed in order to meet timing constraints. Some real-time systems perform treatments that do not depend on the environment, it is then possible to calculate the task schedule *off-line*. This consists in completely determining the execution order of tasks before execution. Then, on-line, the scheduler applies this order.

However, a real-time system must generally respond to external events. Thus, scheduling decisions are taken *on-line*. Much work was carried out in the field of on-line real-time scheduling.

In general, on-line scheduling algorithms are based on priorities assigned to tasks. The ready-to-run tasks are stored in a priority queue, and the highest priority task is executed first. If the scheduling policy is *preemptive*, a task is interrupted as soon as a higher priority task is released; its execution is resumed when no higher priority task is ready. In contrast, when scheduling is *non preemptive*, a task never gets interrupted, and priorities

are exploited only at task termination.

Priorities may be constant during task execution, or changed dynamically. For example the EDF (Earliest Deadline First) algorithm change task priorities dynamically, such that the task with the earliest deadline always has the higher priority.

Other task characteristics can be exploited by the scheduling policy. For example, tasks can be periodic (i.e. be released regularly). The RM (Rate Monotonic) algorithm is a fixed-priority algorithm. RM assigns priorities to tasks depending on their period, the task with the lowest period being assigned the highest priority.

### 2.1.3 Schedulability analysis

A number of *schedulability analysis methods* exist [7]. They determine, for a given task set and scheduling policy, if task deadlines will be met. In this case, the system is said to be *schedulable*.

Some schedulability analysis methods are based on the computation of the *response time* of every task. The response time of a task is the worst-case delay between the task release and the task termination. The response time of a task is equal to the WCET of the task plus the time during which it cannot execute due to processor sharing with the other tasks (i.e. preemptions by higher priority tasks in priority-based on-line scheduling). Schedulability analysis then consists in computing for every task its response time and checking that the task response time is lower than the task deadline.

**Task models** Task models are used to model the system, and more particularly the task release dates (periodic, sporadic, aperiodic) and the relationships between tasks (independent, resource sharing between tasks, precedence relations between tasks). The task model is used by the schedulability analysis method, it is then necessary that the used model is pessimistic as compared to the system behavior itself.

One of the simplest system model is the Liu & Layland model modeled by a set of temporal parameters: the activation period, the worst-case execution time and the deadline. The activation period is the time interval between two successive releases of the same task. The deadline is the delay, starting from the task arrival, before which the task must be terminated. More complex models offer a better modeling of some specific behaviors (e.g. bursts of task arrivals, dependencies between tasks, etc.).

**An example of schedulability analysis method: Response Time Analysis (RTA)** The response time of a task is the worst-case delay between the task release and the task termination. The response time of a task, in the context of preemptive fixed-priority scheduling is the task WCET plus the delay during which the task cannot execute due to preemptions by higher priority tasks.

If the maximum response time of each task is lower than its deadline, then the system is schedulable. The RTA (Response Time Analysis) technique determines the response time maximum of a task.

The RTA technique is based on the concept of *critical instant*. The critical instant is the time point when the task is known to result in its highest response time. For periodic synchronous independent tasks and fixed priority scheduling, the critical instant of a task is when all higher priority tasks are released simultaneously.

J'ai commenc les corrections dans ce sens, mais je suis plus surs du tout, ... The tasks of higher priority will run, then we consider that they are restarted as soon as possible, after activation period. We call "period of activity" the time that makes these tasks to run. The study of the RBF is used to find the length of this period of activity and thus the maximum response time. For a given assignment of priorities, it is possible to determine the response time of each task and to deduce if the system is schedulable.

## 2.2 Real-time systems on multi core architectures

The recent evolution of processors has recently mutated from an increase of the frequency of a single core to an increase of the number of cores. The use of multi-core processors for real-time systems seems unavoidable in the near future. First, performance of these new architectures can meet the performance requirements of the most complex real-time systems. On the other hand, the processor industry tends to focus on multi-core architectures and leave apart the single-core architectures. This mutation to multi-core architectures has motivated many researches in the real-time systems area.

Regarding task scheduling, multi-cores add a spatial dimension to the scheduling problem: scheduling need not only manage the execution of tasks over time but need also manage their distribution between cores.

As far as WCET estimation is concerned, a multi-core processor allows true parallelism between tasks: several tasks are allowed to run simultaneously. However, this raises the problem of shared resources: shared caches, shared busses, etc.. The execution time of a task is different depending on the use of hardware resources by co-running tasks. For example, a task us-



ing running on a multi-core platform with a shared cache behave differently depending on whether or not the co-running tasks make intensive use of the caches. As a result, the WCET of a task depends on the tasks running on other cores. This issue is termed *inter-tasks interference*.

To account for inter-task interference during WCET estimation, ideally one should know the placement of tasks on other cores. This placement is carried out by the scheduler. On the other hand, for checking schedulability it is necessary to know the WCET of tasks. To break this interdependence it's necessary to limit the interferences between tasks. A solution that we detail in Section 2.2.1 is to use cache partitioning.

### 2.2.1 Shared Caches

We have seen that all shared resources within a processor cause inter-task interferences. However, among these shared resources, caches have the biggest influence on the calculation of the WCET. Therefore we will concentrate on caches in the remainder of this document.

There exist methods to estimate WCET for multi-core architectures with shared caches [11] [8]. These methods estimate the worst-case amount of inter-tasks interference due to the shared cache. When possible, some methods even reduce the amount of interferences. However, those methods may lack accuracy, in particular when some tasks have large memory requirements.

**Cache Partitioning** Cache analysis methods for shared-cache multi-cores may lack accuracy, in particular when some tasks have large memory requirements. In such situations, cache partitioning should be preferred.

Rather than accounting for inter-task interference during WCET estimation, cache partitioning methods eliminate such interferences. The shared cache is partitioned (among cores/tasks), and a core/task can only access the partition allocated to it, thus removing all inter-task interferences caused by the shared cache. On the downside, every core/task can only use a subset of a cache, which may result in decreased performance. Different partitioning strategies exist: partitioning between cores, between tasks, with identical or different partition sizes per core/task [12]. In the case of non-preemptive scheduling, since a task is never interrupted once started, it may be worthwhile to allocate a cache partition for all tasks running on the core.

Consider the case of cache partitioning among cores, with possibly different partition sizes each task. The larger the partition size assigned to a task, the lower its WCET. However, for tasks with small memory requirements,

the increase of the assigned partition size beyond a given threshold will not reduce the WCET. The problem consists in finding the most "efficient" allocation. It is therefore necessary to have a metric to evaluate the efficiency of a distribution. The metric used here is the total CPU utilization.

For a task  $i$  with a period  $P_i$  with a WCET  $C_i$ , the CPU utilization  $u_i$  is defined as  $u_i = \frac{C_i}{P_i}$ . Recall that in aperiodic real-time system, the period indicates the minimum time between two revivals of a spot. The maximum CPU utilization is reached when the tasks wake up as often as possible. It is thus possible to define the most efficient cache partitioning as the partitioning that minimizes the CPU utilization. Intuitively, this consist in having the smallest WCET each task, focusing on tasks that run often.

Cache partitioning is similar to the *knapsack problem*, which is a NP-complete problem. Various strategies were therefore defined to make an offline partitioning close to the optimal. An approach with a genetic algorithm is presented in [4]. Other work focus on online partitioning, for example [14].

Other cache partitioning method assigns cache partition to cores instead of tasks. The result is much more interesting because each task have a higher amount of cache. However, with those methods, the interdependance between tasks and cache partitioning remains.

### 2.2.2 Scheduling

The use of multi-core architectures in real-time systems adds a spatial dimension to scheduling, which only had to consider the time dimension in single core architectures. More precisely, when the scheduler run, it have to choose not only which tasks will be executed but also on which core.

Real-time scheduling strategies for multi-cores can be divided into two broad categories: the *global* and *partitioned* scheduling [7]. Partitioned scheduling assigns each task to a single core and prevent task migrations between cores. Under partitioned scheduling, the tasks assigned to one core can be scheduled using mono-core real-time scheduling algorithms; mono-core schedulability analysis methods can be used unmodified. In contrast, global scheduling strategies allow task migrations, at different levels: either at any time during the execution of job, or between the execution of two jobs of the same task.

**Global Scheduling** The global scheduling strategies assign a task to a core when a scheduling decision is taken. This implies that the same task

can be running on different cores throughout its lifetime. In the case of preemptive scheduling algorithms, this implies that task can be started on a core, be interrupted, and then resumed on another core. To limit the overhead of task migrations (migration of the task context, impact of the migration on the cache), some algorithms only allow migrations between jobs of the same task. The first scheduling algorithms for multi-core processors were direct extensions of their mono-core equivalent (e.g. EDF or RM). On a M-core architecture, these algorithms execute the M highest priority tasks on the cores.

However, it was shown that global-EDF or global-RM loose the optimality property they had in a mono-core context. Systems using these algorithms are not schedulable with if the full use of task is only slightly than 1 (i.e. need only slightly more than the capacity of one processor to execute). In a multi-core, it would be more interesting to have a total use closer to the number of processor.

**Partitionned scheduling** Partitioned scheduling assigns each task to a core [2] off-line and are not authorized to migrate between cores at run-time. The advantages of this class of techniques are mainly to avoid migration costs, and to avoid the overhead and synchronization to access a single run queue. Moreover, it is possible to reuse all single-core scheduling theory.

This task partitioning problem is similar to that of the knapsack problem. This is to "fill" available cores with the tasks, such that the task sets assigned to each cores are schedulable. The classical defined for solving the knwpsack problem can be used (first-fit, best-fit, worst-fit).

The WCET is then needed to make scheduling partitioned, as well as for checking the schedulability. Partitioned scheduling is therefore used either after have made a cache partitioning, either ignoring caches.

### 2.3 Joint tasks and cache partitioning

We presented in this section on one hand the cache partitioning technique, and on the other hand partitioned scheduling. By eliminating the interferences related to shared caches, cache partitioning between tasks allows the calculation of WCET without have knowledge of scheduling. However, obtained cache partitioning, which minimize the total CPU utilization may cause the system not schedulable, while the same system with another partitioning would be schedulable. Moreover, it result in small partition assigned to each tasks, and there is a huge part of the cache which is not used at each

moment of execution, i.e. all the cache partitions allocated to tasks which are not running.

A new approach to all those problems proposed by [13], IA3, is to compute cache partitioning and scheduling simultaneously.

**Algorithm IA3** The proposed solution allows partitioning the cache between cores. To avoid interference between the tasks of the same core, scheduling on each core is non-preemptive. IA3 is based on a "WCET-matrix". This is a matrix for one task that indicates the WCET as function of different resources, for example the caches. IA3 uses this matrix to assign more cache to the tasks that are more "sensitive", in a sense that their WCET highly varies when more cache is assigned to them.

**Drawbacks** IA has some drawbacks. First, very few solutions are explored. At some point of the algorithm, IA3 allocates tasks to one core and never returns on this decision. Moreover, while IA uses the WCET-variability of the tasks, there are many other properties of tasks that an algorithm can use to improve the cache repartition. Finally, the fact that it tries to reduce the number of cores makes it non-scalable.

## 3 System model and notations

### 3.1 Architecture

We consider a real time system composed of  $N$  tasks on a multicore processor composed of  $M$  cores which have each a L1 private cache. The processor has a L2 shared cache of size  $S$  which can be divided into  $K$  equalized partitions. The partitioning of the cache is not important, as far as each core can be given any number of partitions between 0 and  $K$ .

### 3.2 Tasks and task scheduling

Each task  $t_i$  has a fixed period noted  $P_i$ . Without loss of generality, we assume  $(i < j)$ . We consider a system with implicit deadline  $D_i, (D_i = P_i)$ . The other properties of tasks, WCET and associated utilization, are not fixed since they depend on the amount of cache that the task has. Instead we define other properties.

**WCET function** The WCET of a task is different according to the amount of cache available at each level of the cache hierarchy. It is possible

to compute the WCET of a task on one core with a given amount of cache in different levels of a cache hierarchy. We therefore define the *WCET functions*  $WCET_i$  for each task  $t_i$  as the follows :

$$\forall n \in [0, K] : C_i^n = WCET_i(n)$$

Where  $C_i^n$  is the WCET of the task  $t_i$  with  $n$  partitions of the shared cache.

**Initial utilization and WCET** Although there are multiple WCET for one task, it's convenient for the algorithm to dispose of a "reference WCET" which would give a good indication of set of WCETs of a task  $t_i$ . Although the WCET functions of the tasks are very different, there are common points between them. In Figure 1 we observe the curve of the WCET of a task  $t_i$  in function of the amount of cache that it dispose of.

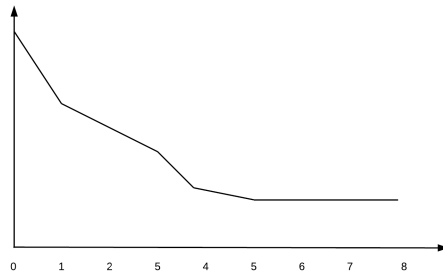


Figure 1: WCET depending of the number of partitions

The first common point between tasks is that there is a huge difference between  $C_i^0$  and  $C_i^1$ . In the most cases the tasks will dispose of at least one cache partition. Then we call  $C_i^1$  the *initial WCET* of the task  $t_i$  and the *initial utilization*  $\frac{C_i^1}{P_i}$ , noted  $U(t_i)$ .

**Optimal number of partitions.** Another common point is that for certain tasks which are sufficiently small, there is a WCET from which adding cache partition has no effect. We call the number of partition corresponding to this amount of cache *optimal number of partition*, noted  $\kappa_i$  defined as the following :

$$C_i^{\kappa_i} = \min\{C_i^k | \forall k' > k, C_i^k \leq C_i^{k'}\}$$

In our example the optimal number of partitions for  $t_i$  is 5.

**Variability.** It's important to determine if the WCET of a task varies greatly or little when allocating more cache to it. The variations of WCET can be very different depending of the nature of task. In consequence we define an approximation of the WCET variability. We call *variability* of a task  $t_i$ , noted  $V(t_i)$ , the absolute value of gradient of the line between its initial WCET and its optimal WCET. Formally :

$$V(t_i) = \left| \frac{C_i^{\kappa_i} - C_i^1}{\kappa_i - 1} \right|$$

The *normalized variability* rank is the rank of a tasks in the task set sorted by variability, divided by the total number of tasks. Thus, a normalized variability rank close to 1 indicate a task part of the tasks with the lowest variability of the task set.

**Task scheduling** In this article we focus on non-preemptive system. Non-preemptive systems are simpler when the caches are taken into account. When a task is preempted, the new tasks can potentially evict all the cache entries of the private L1 cache and the partition of the shared L2 cache. Moreover, non-preemptive systems are actually mainly used in the industry, for example in the aerospace. Therefore, we consider that the system is non-preemptive and scheduled by NP-EDF.

The system is non-pre-emptive and scheduled by NP-EDF.

### 3.3 Problem formalization

The considered joint tasks and cache partitioning problem consists in assigning each task to one core and partitioning the cache among cores, such that the resulting system is schedulable under the algorithm *non-preemptive EDF*.

Formally, from a task set of  $N$  tasks,  $M$  cores, and  $K$  partitions, the aim is to build a configuration *valid* and *schedulable*. A configuration is a set of  $M$  pairs  $(\tau_c, k_c)$  where  $\tau_c$  is a set of the tasks assigned to core  $c$  and  $k_c$  is the number of partition assigned to core  $c$ . We call a configuration valid if (i) it map every task to one and only one core and (ii) the total number of partitions assigned to cores is less than or equal to  $K$ . We call a configuration schedulable if the system in which the tasks and cache are partitioned as describe by the configuration is schedulable under non-preemptive EDF.

**configuration schedulability** In a valid configuration each task is assigned to a core which has a fixed number of partitions. Therefore, the

WCET of each task  $t_i$  is known and equal to  $C_i^k$ . For a core  $(\tau_c, k_c)$  the WCET of each task  $t_i \in \tau$  is  $C_i^k = WCET_i(k)$ . When obvious from context, we use the notation  $C_i$  instead.

A core  $(\tau, k)$  is schedulable with NP-EDF iff it satisfies two conditions [10]:

$$\sum_{t_i \in \tau} \frac{C_i}{P_i} \leq 1 \quad (1)$$

$$\forall t_i \in \tau, \forall L, P_1 < L < P_i : L \geq C_i + \sum_{j=1}^{i-1} \lfloor \frac{L-1}{P_j} \rfloor C_j \quad (2)$$

The configuration is schedulable if the taskset of each core is schedulable.

## 4 The algorithm

Our proposition, PDAA, Period Driven Assignment Algorithm, is an algorithm which aims at finding a solution to the joint tasks and cache partitioning problem. As previously seen with the algorithm Non pre-emptive EDF, the difference of periods between the tasks mapped on the same core is critical. Hence, PDA will assign tasks to cores depending of their period, then try to explore different configuration to minimize the total utilization, in order to find a schedulable configuration.

### 4.1 Rationale

#### 4.1.1 New schedulability condition and relative weight.

From condition 1, we clearly see that our algorithm will have to deal with the total utilization of the differents cores to obtain a schedulable configuration. However, condition 2 does not make appear what kind of configuration could satisfy it.

In consequence, we will instead consider a new schedulability condition, that have been shown a necessary but not sufficient condition for condition (2) [1]:

$$\forall t_i \in \tau : C_i + \sum_{j=1}^{i-1} C_j + u_j(P_i - P_j) < P_i \quad (3)$$

The algorithm tries to build configuration which satisfy this new condition. However, when the schedulability of a configuration is actually tested, the condition used are condition 1 and 2.

The important part of the equation is the term  $u_j(P_i - P_j)$ . We call this the *relative weight* of the task  $t_j$  for the task  $t_i$ . With condition 3 we clearly see that the aim is that for each task  $t$ , minimize the sum of the relative weights for  $t$  of the tasks with a lower period than  $t$ . We see also that the more the period of  $t$  is, the bigger this sum can be.

#### 4.1.2 Task placement

Recall that for the computation of the sum of relative weights we consider only the tasks with a lower period. We therefore can say that a task with a big utilization have to be one the tasks which have the biggest period of the core. In this manner the big utilization of the task is counted only for the few tasks which have a bigger period.

Moreover, in order to reduce the relative weight of tasks, we want the difference between period of tasks minimized, or that the utilization is small. Then, the tasks which have the bigger difference of period with the tasks that have the bigger period of a core must have small utilization.

A task assignment which respect the previous directive is presented in figure 2. We've seen that in the returned configuration the tasks with the biggest utilization have to be the ones with the biggest period of the core.

The biggest tasks will actually be chosen having a big initial utilization and a low variability. Thus, whatever the partition among cache is, those tasks will always have a big utilization. Since condition 3 shows that two tasks with a very small difference of period have a small impact, even with a big utilization, the biggest tasks of all core are chosen in such a manner that their periods are distributed among the period interval of the taskset. Those tasks will be determinant for the placement of the other tasks. We call them *critical tasks*. Those tasks are represented by a big circle on the example.

As the critical tasks are considered the biggest of one core in terms of utilization, we don't want that other tasks of greater period assigned to the core. Therefore the criticals tasks reduce the possibilities of placement of the other tasks. Thus a given task can only be assigned to a core which has a critical task of greater period than its period. On the example, each non critical task is represented by a white circle, and the period of critical task is marked with a vertical pointed line. On this example each task has to be



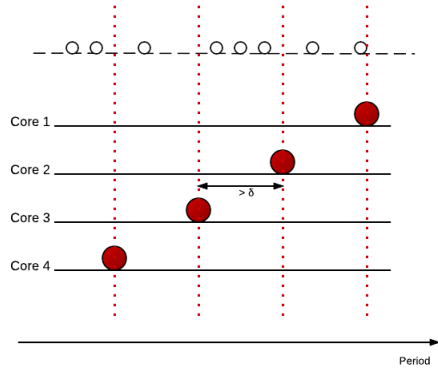


Figure 2: Placement of critical tasks

at the left of a critical task, therefore the possible cores are the ones with a pointed line righter than the task.

The placement which minimizes the difference of periods is when each task is assigned to the core which have the *critical task* with the period the closest to the task. On the example, such a placement is obtained when tasks are assigned to the lowest possible core.

However, such a placement will possibly result in cores with many tasks which will not be schedulable and cores with very few tasks. Moreover, finally, as a smaller utilization give a smaller relative pound, we want a placement which takes advantage of the optimal number of partitions and variability of task in order to optimize the cache assignment and therefore reduce the utilization of each task. More precisely, the tasks with a close optimal number of partition should be on the same core. However as it's often not possible that all tasks have a number of partition close to the optimal, the tasks with a big variability are prioritized.

Therefore, our algorithm will "bring up" some tasks from the unschedulable cores, taking account into the initial utilization and variability of the tasks as described.

## 4.2 The algorithm

The main steps of the algorithm are presented in algorithm 1. It takes as parameters the set  $\tau$ . PDAA is divided in three main phases.

The first phase consists in obtaining the placement where all tasks are assigned to the lowest possible core.

First the critical tasks are determined, assigned to the cores, and then removed from the tasks set. Hence  $\phi'$  is the configuration with the critical tasks assigned, and  $\tau'$  the task set without the critical tasks. These two sets are fixed and not modified afterward. Second the other tasks are assigned in such a way to reduce the difference of their period and the periods of already placed tasks. Then, the `allocate_cache_min` function is invoked.

---

**Algorithm 1**

---

```

function PDAA
   $\phi \leftarrow \emptyset$ 
   $(\phi', \tau') \leftarrow \text{PLACE\_BIGGER\_TASK}(\tau)$ 
   $\phi \leftarrow \text{PLACE\_OTHER\_TASKS}(\tau', \phi')$ 
   $\phi \leftarrow \text{ASSIGN\_CACHE\_MIN}(\phi)$ 
  while  $\neg \text{IS\_SCHEDULABLE}(\phi) \wedge i < nbTry$  do
     $\phi \leftarrow \text{MOVE\_TASKS}(\tau', \phi', \phi)$ 
     $\phi \leftarrow \text{ASSIGN\_CACHE\_MIN}(\phi)$ 
     $i \leftarrow i + 1$ 
  end while
  if  $\text{IS\_SCHEDULABLE}(\phi)$  then
     $\phi \leftarrow \text{ASSIGN\_TOTAL\_CACHE}(\phi)$ 
    return  $\phi$ 
  end if
  return  $\emptyset$ 
end function

```

---

The aim of `assign_cache_min` function is to determine if there exists an assignment of cache partitions for which the configuration is schedulable. The function assigns the minimal number of cache partitions to a core for which it is schedulable. Hence the sum of minimal number of cache partition of all core might be greater than  $K$ , when there is no cache partition which allows the configuration to be schedulable and the tasks have to be moved.

During the second phase different solutions are explored. While the current configuration is not schedulable, the algorithm tries to bring up some tasks taking account the variability and initial utilization. Then, the `assign_cache_min` is invoked again to check if there exist a schedulable configuration. If not, tasks are moved again. The number of time this loop is executed ( $nbTry$ ) is a parameter of the algorithm, that will be discussed in section 5.

When a schedulable configuration is found, the `assign_total_cache` function assigns all the cache partitions with a greedy algorithm in such a way that the

number of partitions allocated per core is greater than its minimal number of partition.

If no schedulable congiration is found, the algorithm return  $\emptyset$ .

## Phase 1

**Identification and placement of critical tasks.** The first step of the algorithm is to assign the critical tasks to the cores. It's done in the function `place_critical_task` presented in algorithm 2. The critical tasks are those which have a big initial utilization and a small variability. Thus, the tasks are sorted by utilization, and taken in decreasing order.

For each tasks, the normalized variability rank is computed.

If the normalized variability rank is bigger than a given threshold, then the tasks is considered to be part of the biggest task of the tasks set, and will be assigned to a core.

---

### Algorithm 2

---

```

function PLACE_CRITICAL_TASKS( $\tau$ )
   $\phi \leftarrow \emptyset$ 
   $\tau' \leftarrow \tau$ 
  threshold  $\leftarrow 0.9$ 
  while  $\text{card}(\phi) \leq M \wedge \textit{threshold} > 0$  do
    for all  $t \in \tau'$  by decreasing utilization do
       $v \leftarrow \text{NORMALIZED\_VARIABLY\_RANK}(t, \tau)$ 
      if  $v > \textit{threshold}$  then
        if  $\neg(\exists(\tau, y) \in \phi, t' \in \tau \wedge |P(t) - P(t')| < \delta) \wedge \text{card}(\phi) \leq M$  then
           $\tau_i = \{t\}$ 
           $\tau' \leftarrow \tau' - \{t\}$ 
           $\phi \leftarrow \phi \cup (\tau_i, 0)$ 
        end if
      end if
    end for
    threshold  $\leftarrow \textit{threshold} - 0.1$ 
  end while
  return  $(\phi, \tau')$ 
end function

```

---

As previously said, the period of critical tasks of each core has to be far from each other. Then if the chosen tasks have a period close to a biggest task already assigned, then it assigned to this core. Else, it's assigned to a new core. Two tasks are considered close if the difference of their period is lower than  $\delta$ . This process is repeated until there is no more tasks in the

taskset. If all the cores have a critical task, then the function returns. Else, the threshold is decreased and the whole process repeated with the remaining tasks.

Once the biggest tasks are assigned the configuration is stocked in  $\phi'$ . In the following of the algorithm,  $\phi'$  will never be modified. In contrary, the other tasks can be moved and a total configuration at any point of the algorithm is stored in  $\phi$ .

**placement of other tasks** An initial placement is achieved by function by *place\_other\_tasks*. Since the tasks must not be assigned to a core with a critical task with a lower period than it, the possible cores for a task are determined by its period. The first placement is such that the difference between period is minimized. Hence each task is assigned to the core which have the critical tasks with the minimal period greater than its. This is the configuration in which the tasks are at the lowest possible core.

**Cache assignement** The cache min assignment is presented in algorithm 3. This function is quite simple. For each core the function assigns a number of partitions, then test if the core is schedulable. If not, the number of partitions is increased by 1. If it is, the minimum numbers of partitions is reached. If a core is not schedulable with  $K$  partitions, then the number of partitions assigned is  $K + 1$ . Thus, if the total amount of number of partitions is lesser than  $K$ , the configuration is schedulable.

---

### Algorithm 3

---

```

function ASSIGN_CACHE_MIN( $\phi$ )
  for all  $c = (\tau, y) \in \phi$  do
     $i \leftarrow 0$ 
     $c \leftarrow (\tau, i)$ 
    while  $\neg$  IS_CORE_SCHEDULABLE( $c$ )  $\wedge i \leq K + 1$  do
       $i \leftarrow i + 1$ 
       $c \leftarrow (\tau, i)$ 
    end while
  end for
end function

```

---

### phase 2

**Moving Task** The function Move\_task is presented in algorithm 4. Since the place\_other\_task function assign the task at the lowest possible

core, the `move_other_task` function will push some tasks up. With the result of `allocate_task_min`, it's possible to know which cores are not schedulable (those with a number of partitions greater than  $K$ ).

For each unschedulable core the task that will be pushed up is chosen by the function `max_score_task`. The aim is that after the move, the total utilization of the core is highly decreased. Thus, the task chosen must have a big initial utilization. After the move, the core which will get the task must not increase its utilization too much. Since this core will be chosen in a such manner that the already assigned task will have a optimal number of partition close to the moving task, the moving task is chosen with also a big variability. The score of a task  $t$  is then  $\alpha U(t) + \beta V(t)$ . The task with the maximal score is chosen to be moved.  $\alpha$  and  $\beta$  will be determined empirically.

Next, the set of core "upper" than the considered core is computed by `get_upper_core`. The core chosen is the core with the minimal difference between the average optimal number of partition of assigned tasks and the optimal number of partition of the moving task.

---

**Algorithm 4**

---

```

function MOVE_TASK( $\tau', \phi$ )
  for all  $c = (\tau_c, k_c) \in \phi$  do
    if  $k_i = K + 1$  then
       $C \leftarrow$  GET_UPPER_CORES( $c$ )
       $t \leftarrow$  MAX_SCORE_TASK( $\tau_i$ )
       $c' \leftarrow$  MAX_CORE_AFFINITY( $C, t$ )
      MOVE_TASK( $t, c, c'$ )
    end if
  end for
end function

```

---

**phase 3** The total cache assignment tries to reduce the utilization of a schedulable configuration. At the beginning,  $K$  partitions are assigned to each cores. Then, for each core if the minimal number of partitions is not reached, the total utilization with a the number of partitions minus one is computed. the core with the biggest utilization reduction has its number of partitions assigned decrease by one. This process is repeated until  $K$  partition are assigned.

## 5 Experimental results

We have implemented PDAA and IA3 and compared the two algorithms on synthetic tasksets. We present in this section the performance metric for the evaluation algorithm, how the tasks are generated, the determination of empirical results of PDAA, and finally a comparison between the two algorithms.

### 5.1 Methodology

#### 5.1.1 Performance metric

In order to compare the algorithms, we have to define a metric of their performance. We will use the percentage of schedulable configurations of an algorithm among the tasksets generated. We then want to see the evolution of this metric in function of variations of parameters. Every experiment will report the average of 100 task sets.

#### 5.1.2 Tasksets generation

The task set generation process consists in generating  $N$  tasks with a total utilization  $U_{sum}$ .  $U_{sum}$  is the sum of the worst utilization of the tasks. The worst utilization is the utilization of the tasks without any cache available. At the end of the partitioning the total utilization is then largely lower than  $U_{sum}$  since each task will have some cache space reserved. In consequence we will test our algorithm with tasks with  $U_{sum} > M$ .

Bini et al. [3] have shown that the utilization for tasks have to follow a continuous uniform law, in order that the test is not biased. They also provide an efficient algorithm to generate those utilizations. We'll then use it to obtain the worst utilization of the tasks noted  $U_i^0$ .

For a task  $t_i$  we can compute the worst WCET  $C_i^0 = U_i^0 \times P_i$ . With a coefficient  $c$  picked randomly between 0.2 and 0.4, the best WCET is  $C_i^{\kappa_i} = cC_i^0$ . The number of partitions of partition  $\kappa_i$  is picked randomly between the lowest and the highest optimal number of partition which are part of the parameters. Finally, the others WCET of  $t_i$  are determined: the WCET for number of partition greater than the optimal number of partition are equal to  $C_i^{\kappa_i}$ . Other WCET are on the line  $((C_i^0, 0), (C_i^{\kappa_i}, \kappa_i))$ .

## 5.2 Determination of the parameters of PDAA

### 5.2.1 Choice of fixed parameter

For the evaluation of the performance of our algorithm, we'll measure the performances in function of parameters of the input or of the systems. More precisely, the parameter of the input task set are the total utilization, the number of tasks, the lowest and the highest number of partitions. The parameter of the system that we'll make vary is the number of cores, the amount of cache and the number of partitions. In a first time we'll chose definitive value for some parameters, then, we'll run experiments to determine the value of the parameter of the algorithm.

**Lowest and highest optimal number of partitions.** Preliminary experiment have shown that the lowest and highest optimal number of partitions have no effects on the performance of the algorithms. Consequently, for the experiments presented in this section, we'll chose for the lowest number of partition the number of partition which correspond to a size equal to  $\frac{S}{128}$  and for the highest the number which correspond to  $\frac{S}{2}$ .

**Number of partitions.** We then run different tests for a fixed load on differents numbers of cores and differents numbers of tasks. Figure 3,4 show the evolution of the efficiency of PDAA for different numbers of tasks on respectively 2 and 4 cores. Results for 6 and 8 cores are similar.

We see that under a certain number of partitions, the algorithm is not able to compute schedulable configurations. However, over a certain number of partitions, increasing the number of partitions does not increase the efficiency of the algorithm. Thus, we'll use the value 10 for all the following experimentats.

**Cache size.** Next, we run tests in order to determine the impact of the cache size on the percentage of the schedulable configuration returned. As presented previously, Figure 5,6 presents the results. The amount of cache make no difference here, thus we chose a typical cache size of 128KB.

### 5.2.2 Empirical parameters

With the parameters determined above, we want to choose value for the empirical parameters used by PDAA. Recall that we have four empirical parameters:

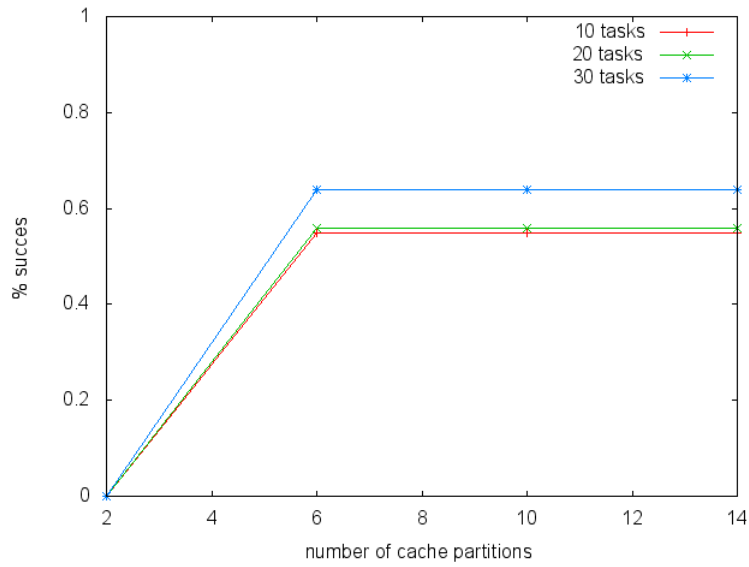


Figure 3: percentage succes for different numbers of partition with 2 cores

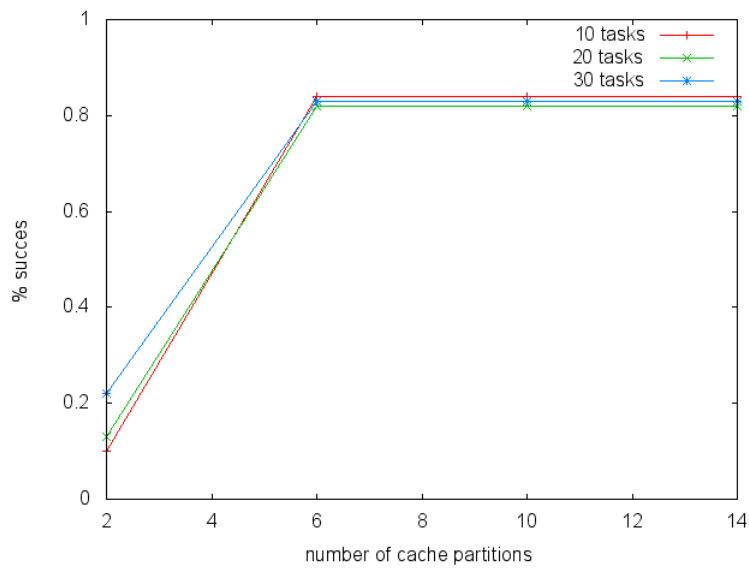


Figure 4: percentage succes for different numbers of partition with 4 cores

- the minimal difference of period between critical tasks :  $\delta$



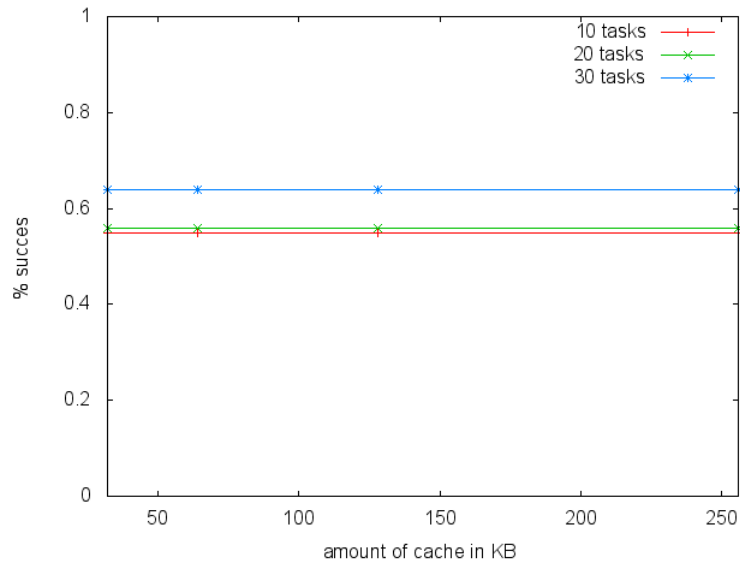


Figure 5: percentage succes for different cache sizes with 2 cores

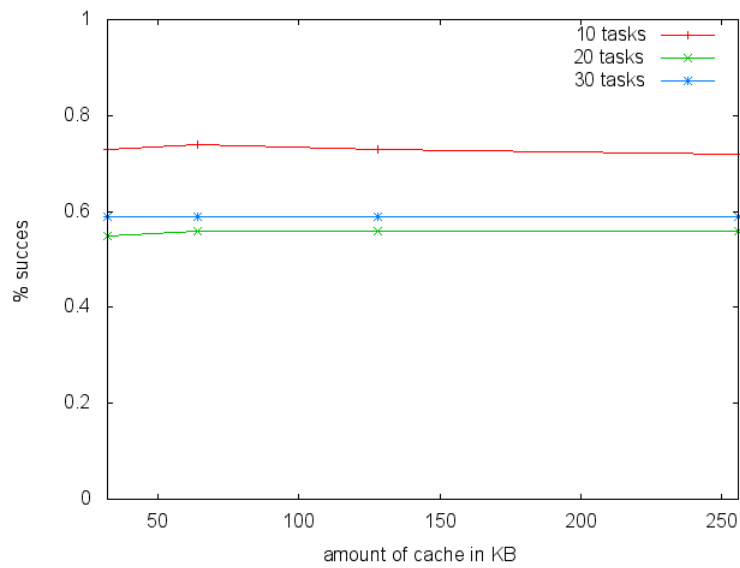


Figure 6: percentage of succes for different cache sizes with 4 cores

- the two coefficients for the score of tasks :  $\alpha$  and  $\beta$

- the number of attempts to re-allocate a task before giving up returning a schedulable configuration :  $nbTry$

First, we set  $nbTry$  to a big value ( $2N$ ), so that it will no interfere with other experiment. Then, we'll run tests to determine the other parameter. Finally, we reduce  $nbTry$  in a such manner that a greater value value makes few difference in the performance.

**Delta** The parameter  $\delta$  as presented previously is actually composed of a fixed part that we will not make vary. Precisely, if  $P_{min}$  is the minimal period of the input taskset and  $P_{max}$  the maximal period,  $\delta$  is defined such that :

$$\delta = \frac{P_{min} - P_{max}}{M} \times \frac{\Delta}{100}$$

Thus, the interval of the periods of the input task set is divided by  $M$ .  $\delta$  is actually  $\Delta\%$  of the result. What will be determined empirically is  $\Delta$ .

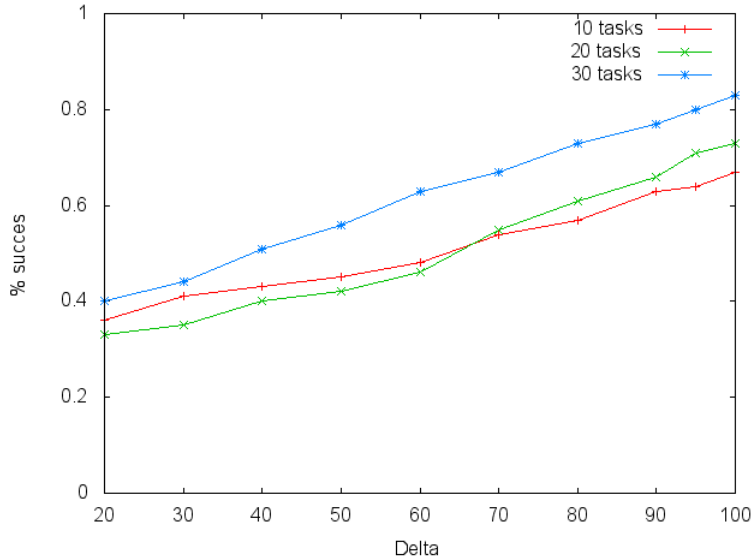


Figure 7: percentage of succes for different values of  $\Delta$  on 2 cores

Figure 7, 8, 9 present the result for 2,4 and 6 cores. We see that the efficiency depends largely on this parameter. The efficiency is maximized for almost every test presented when  $\Delta = 90$ . We thus choose this value.

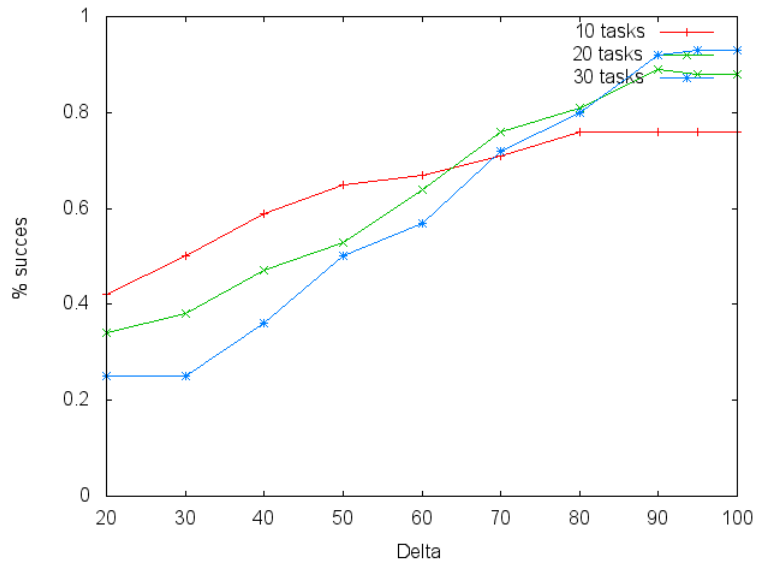


Figure 8: percentage of succes for different values of  $\Delta$  on 4 cores

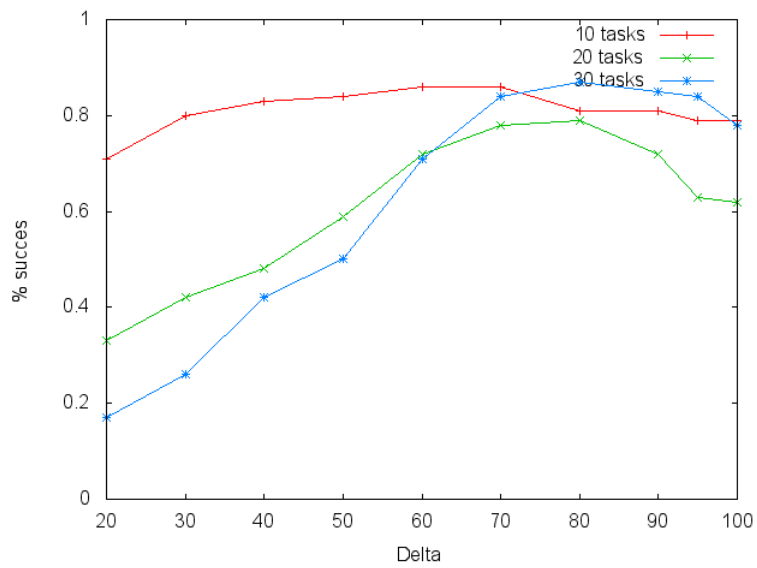


Figure 9: percentage of succes for different values of  $\Delta$  on 6 cores

$\alpha$  and  $\beta$  Because of time constraints, there is no experiments yet for the determination of  $\alpha$  and  $\beta$ . Therefore, there are arbitrarily fixed to 1. Thoses

parameters can only have influence on the choice of the tasks moved during the displacement part of the algorithm. Therefore we can only expect few improvements with better values.

**NbTry** Finally, we ran the experiments with different value of the *nbTry* parameter. For the previous experiment, we had  $nbTry = 2N$ . We tested with value  $N$ ,  $N/2$  and  $N/3$ . This made absolutely no difference in every experiment that we made, so we chose the lowest value :  $nbTry = N/3$ .

### 5.3 Comparison with IA3

Finally, we run experiment in order to determine if PDAA is more efficient than IA3 in terms of percentages of schedulable tasksets. We ran three experiments. First, for a given number of cores and a given number of tasks, we evaluate the efficiency in function of the total utilization. Then, we present experiments that describe the efficiency in function of the number of tasks. Finally, we make vary the number of core, in order to see if PDAA could take benefit of more cores in order to improve the efficiency.

**Load** Figures 10, 11, 12 present the percentage of schedulable configuration in function of the total utilization of the taskset.

We can observe that PDAA is better than IA3, especially on system with 4 and 6 cores.

**number of tasks** Figures 13, 14 present the percentage of schedulable configuration in function of the number of task for a fixed total utilization.

We see that the number of task has little influence of the percentage of schedulable configuration. We can notice a for both PDAA and IA3 a small improvement when the number of task is increased. We explain that by the fact that with more task for a given total utilization, tasks will trend to have smaller utilizations and then the algorithms can distribute the load among core more finely.

**number of core (scalability)** Finally, we test the scalability of the algorithm. We measured the efficiency metric in function of the number of core, for 20 tasks. Figure 15 and 16 present the result for a total utilization equal to respectively 6 and 9.

We clearly see that PDAA take benefit of more core while IA3 is non-scalable.

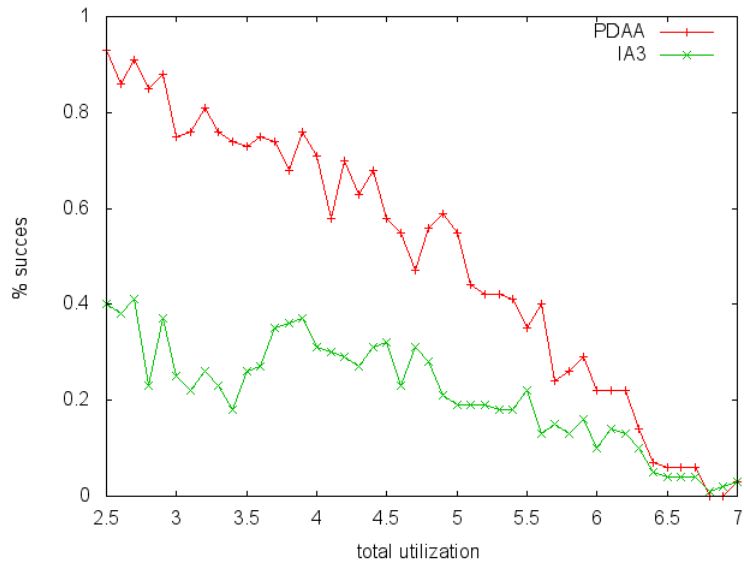


Figure 10: percentage of succes for different values of total utilization on 2 cores

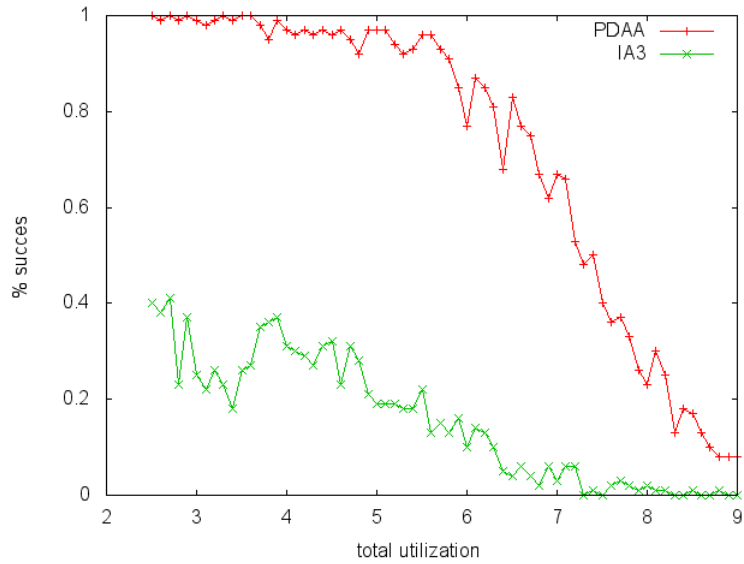


Figure 11: percentage of succes for different values of total utilization on 4 cores

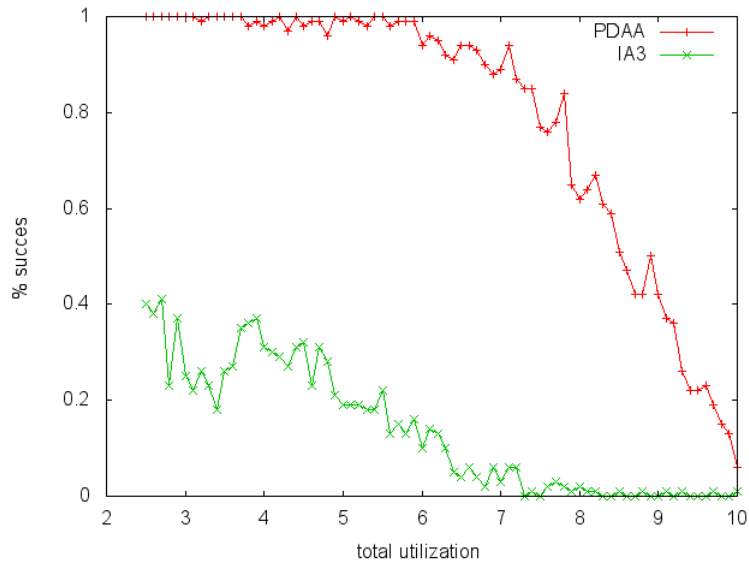


Figure 12: percentage of succes for different values of total utilization on 6 cores

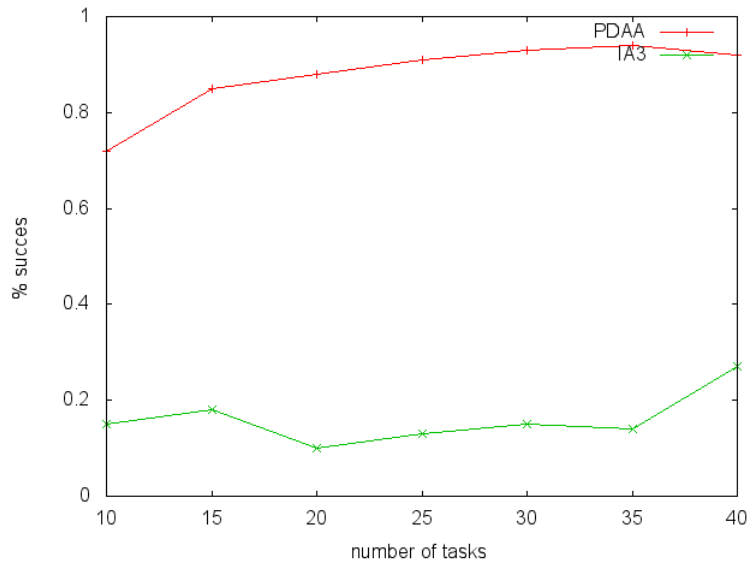


Figure 13: percentage of succes for different number of tasks on 4 cores

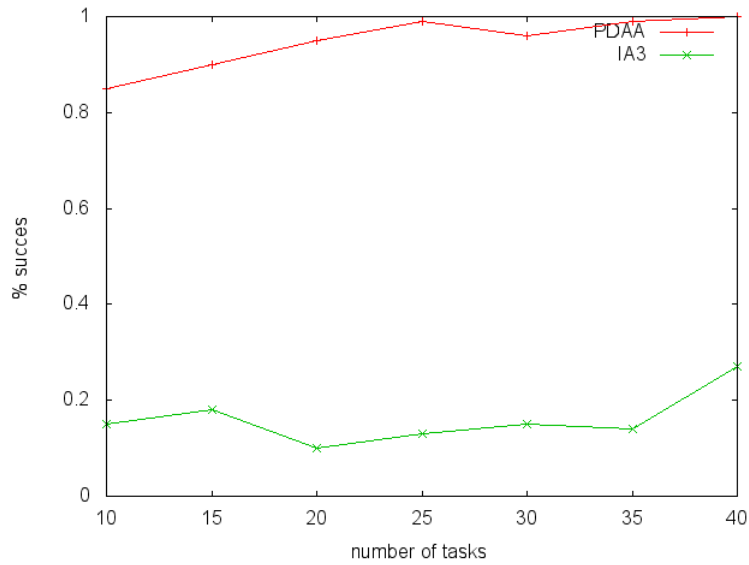


Figure 14: percentage of succes for different number of tasks on 6 cores

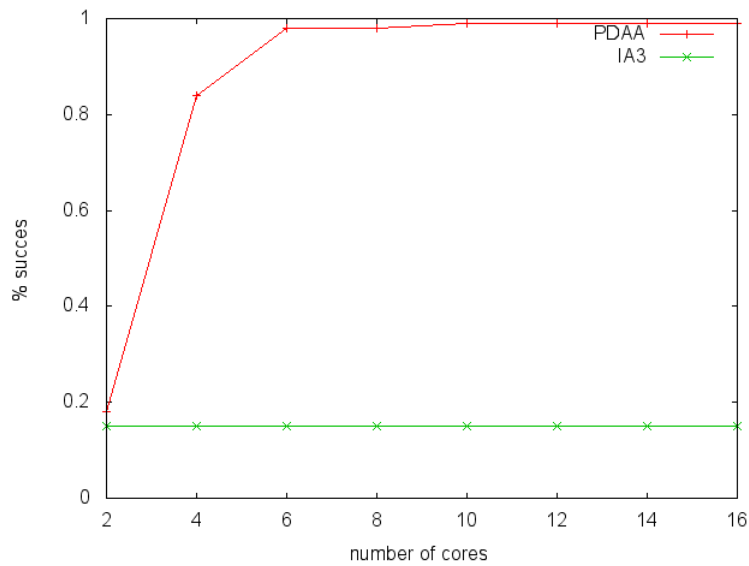


Figure 15: percentage of succes for different numbers of cores with  $U_{sum} = 6$

## 6 Conclusion and future work

In this document we have presented PDAA, a new algorithm for the joint tasks and caches partitioning problem. It outperforms the state of the art

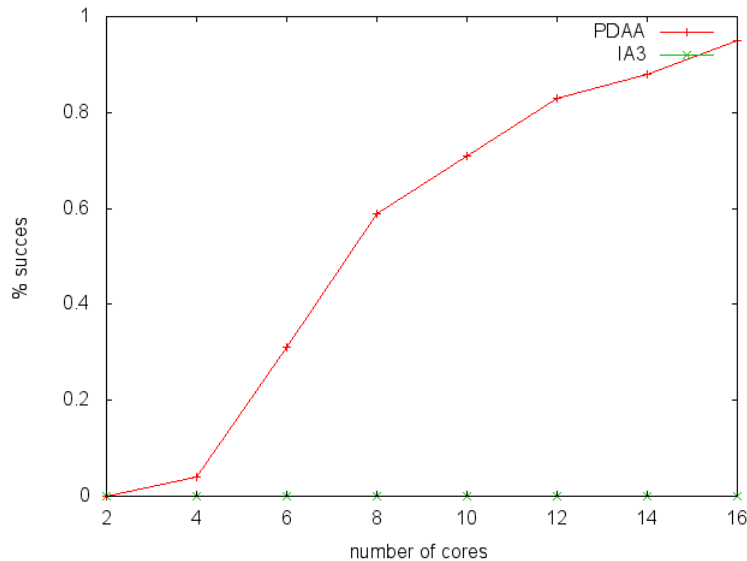


Figure 16: percentage of succes for different numbers of cores with  $U_{sum} = 9$

in terms of our metric of efficiency. This result is obtained by taking into account the repartition of the periods of tasks of one core, which is critical in a system scheduled by NP-EDF. Moreover, PDAA is much more scalable than IA3, since it takes benefit of all core instead of trying to reduce the number of core used.

However, there are multiple possible improvements. First, we plan to extend the algorithm to work with a full cache hierarchy, not only with the L2 cache. Another possibility is to search for a preemptive algorithm. Finally, an important future work is to improve the representativity of the evaluation by experimenting on a real application and validate the task set generation.

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