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To cite this version:
Guillaume Claveres. Tax competition and provision of federal public goods. Economies and finances. 2014. <dumas-01108379>
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sous la direction de Agnès Benassy-Quéré

Présenté et soutenu par Guillaume Claveres — 2014
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Tax competition and provision of federal public goods

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June 2014

Abstract

Models of tax competition can include various features to amend the key conclusion of the canonical literature that states that capital mobility leads to the under-provision of public goods. This paper investigates if that result still holds once shared public goods are introduced, relating to concepts present in the fiscal federalism literature. This is particularly relevant if we consider the case of the European Union that develops federal wide public goods and policies, while having its members countries competing fiscally.

We propose a model with different tax bases at the local and the federal levels and show that sharing the public good can restore optimum. Second, we study how this is not possible in a model with overlapping tax bases. We also present a model with transfers.
1 Introduction

As a sign that they are crucial parameters to understand the provision of public goods and the effectiveness of fiscal systems, base erosion and fiscal incentives have become over the past few years a high profile topic for policy makers and media\(^1\). Indeed, governments are becoming increasingly concerned about the lack of an unified international framework when it comes to corporate taxation, afraid of losing tax revenue to other countries and hence tempted to protect themselves from base erosion with unilateral measures (OECD, 2013a). Concerns over a harmful tax competition and the lack of coordination have been voiced by the European Commission (1997) and the OECD (1998, 2000). Namely, issues include falling corporate income taxes that could lead to loss in revenue, and potential shifts of the tax burden on labor that could create unemployment. Hence, the G20 finance ministers asked the OECD to develop a set of guidelines designed to address those issues, known as the Action Plan on Base Erosion and Profit Shifting (BEPS). Such work of the OECD provides us with some data on base erosion. Although it is impossible to assess with certainty the extent of BEPS behaviors, the organization states that they are widespread. Data on corporate income tax rates draws a downward trend, with a statutory rate in OECD countries that dropped from 32.6 to 25.4% on average over the period 2000-2011 (OECD, 2013b). There were tax rate cuts in 31 OECD countries, a trend that seemed to be initiated by a series of tax reforms in the United Kingdom and the United States during the 1980’s.

The literature on tax competition, displayed in section 2, is extensive and its key conclusion is that capital mobility, by eroding the tax base when a tax rate increases, leads to downward pressure on tax rates and an under-provision of public goods. The goal of this

\(^1\) see « The Great Corporate Tax Dodge » by Bloomberg, http://topics.bloomberg.com/the-great-corporate-tax-dodge/ for further work
paper is to shed new light on those interactions by setting up a model that includes an unprecedented feature: a shared public good. Indeed, there is a lack of literature that attempts at explaining the provision of federation wide public goods.

Regarding the case of the European Union, the importance of such goods have been highlighted on several occasions. National submissions to EU budget reviews include the following: «If the EU budget is to generate the added value called for by the European Council, the structure of the budget must be revised to enable the Union to focus on supporting growth, competitiveness, expertise and innovations in policy areas where it is able to operate more effectively than the Member States, and to produce European level public goods, such as internal and external security and protection of the environment.» (European Commission, 2007-08). So far, policies at the EU level include namely agriculture, with the Common Agriculture Policy accounting for 46.7% of the total budget in 2006, expected to decrease to 32% in 2013. Moreover, expenditures include regional support to poorer regions through structural funds representing 30.4% of total budget in 2006, expected to rise up to 36% in 2013. Finally, internal policies (education, youth, culture, Trans-European networks, environment), represented around 8.5% of total budget in 2006. Expenditures at the EU level are funded by raising revenue on own resources sources, the largest contribution coming from GNI-based resources followed by VAT-based ones. Regarding the former, European citizens are taxed portion of VAT levied in each member country at various rates, although steps are being taken to harmonize the overall tax base. Anyway, the EU budget represented a total of 864.3 billion euros for the period 2007-2013, amounting only to 1.05% of the EU-27’s GNI (forecast). We are far


3 «EU budget facts and myths», EU press release, 2007

from the 16.7% GDP that represented the federal tax revenues in the United States in 2013⁵.

Those figures show how low is the EU budget, raising the question of the under-provision of federation wide public goods, and this is where the theory on fiscal federalism comes into play. It studies, as exposed in the next section, how expenditure functions and fiscal instruments are centralized and decentralized across different levels of government. We will make use of the different interactions that exist vertically (between the local and federal layers of government) and horizontally (between the jurisdictions) that stem from those allocations to study what could be the impacts of tax competition and base erosion on the provision of federal goods financed by several countries, just as in the EU.

Our paper consists in extending a baseline model of tax competition inspired by Zodrow and Mieszkowski (1986) and Wildasin (1988) by augmenting it with a federal layer: a second public good, federal and shared among the different countries. In this new setting, two countries form a federation, each having an independent government that decides its local tax rate to finance its local non shared good and its federal rate to finance the federal shared good that both enjoy. This in itself is original: most of the models of tax competition assume competitive local government and a centralized federal government. We use a federation of countries, each setting independently their tax rates as strategic variables, to maximize the utility of their respective representative citizen. As an example, we can think of an environmental tax that is decided by each country independently and that finances collectively policies on environmental protection, considered as a shared federal public good. We consider cases of autarky, in which tax bases cannot erode in reaction to a change tax rates, leading to an efficient allocation of resources. In other terms, marginal utilities obtained from all sources (private, local and federal public

⁵ Monthly Budget Review—Summary for Fiscal Year 2013, Congressional Budget Office, 2006
consumptions) are equalized in a first best optimum situation. Then, opening up economies makes tax bases mobile between the two members which engage in a competition for those resources as they can flow in the country that offers the better after tax return. This, and this is the key intuition of the canonical tax competition literature, is supposed to lead to a « race to the bottom » in tax rates and to an under-provision of public goods. Our paper investigates if that result still holds in a situation where the public good is shared across the federation.

First, we will study a case in which the two public goods are financed through source taxation of different bases. We observe that sharing the financing of the public good lowers the incentives for the members to undercut each other, reducing the cost of losing tax base to the other country. As a result, the under-provision is reduced compared to a situation in which the good is not shared, and there can even be over-provision. Even more so, if federal tax rates are equalized across the federation, efficiency in the provision of that good is restored.

Second, we turn to a case in which both public goods are financed through taxation of the same base. This is particularly relevant in the EU if we consider VAT taxes that exist nationally and at the federal level to finance the EU budget. We find that this produces adverse effects in the provision of both public goods. Indeed, and this is another result of our paper, overlapping tax bases for two layers of government can magnify the negative impact of base erosion and prevent the federation from providing an efficient level of federal public good for both countries at the same time. This happens even if countries manage to harmonize their rates at the federal level. This result of our model is consistent with the literature on vertical fiscal externalities in federations, that states that overlapping tax bases across different layers of government lead to non-optimal decisions (Dalhby, 2000, 2003).
Finally, we offer an extension to our model that consists in introducing transfers. We show that in this setting increasing transfers reduces the inefficiencies for the recipient country at the expense of the other.

The plan of the paper is the following: a brief literature review is offered in section 2. The model is developed in section 3. An extension to that model is presented in section 4. Concluding comments are made in section 5.

2 Literature review

Our analysis combines segments of the literature on fiscal federalism and on tax competition.

Weingast (2007) displays the characteristics of the two generations of literature regarding fiscal federalism. The first (FGFF) is normative and assumes benevolent social planners. It is pioneered by Musgrave (1959) and Oates (1972) and introduces the importance of intergovernmental interactions at the vertical and horizontal levels. Vertical imbalances arise when the central layer of the government is better at levying taxes than lower levels, while horizontal ones appear if income diverges among same level government entities, leading to a difference in their ability to collect tax revenue. FGFF investigates the instruments that could potentially correct those imbalances, such as transfers or equalization grants (Boadway et al., 1982). Although they will not take this form in our paper, vertical and horizontal effects will arise in our model and their prime importance in the design of tax systems will be reflected. The second generation of fiscal federalism literature (SGFF) builds on FGFF while including political choice environments and the fact that lower level governments have incentives of their own (Oates, 2005). More positive, SGFF includes a large array of literature. To an extent, SGFF will provide less insights
than FGFF as we do not include much political choice features in our model except when it comes to defining an institutional framework allowing for horizontal equalization.

Our model of tax competition is inspired by the synthesized model of Krogstrup (2004), that we will present in the next section as our reference model. Indeed, it offers a tractable setting that we can easily augment while preserving the key features of tax competition. It takes over the seminal papers of Zodrow and Mieszkowski (1986) and of Wildasin (1988). Those authors formalize models of tax competition in which national governments set tax rates to attract an internationally mobile tax base to finance a public good. The well known result of that literature is that countries will compete in tax rates by undercutting each other, distorting downward the provision of the public good. Indeed, countries engage in a one shot game in which capital can flow in and out in reaction to a change in tax rate, to regain the exogenous international after-tax return. Hence, the core concept, mentioned above, is base erosion: increasing a rate causes the tax base to shrink as it flows out to other countries, creating an externality that distorts the provision of the public financed by that tax base.

Such a result is put in perspective by allowing less restrictive sets of assumptions, in papers mentioned by Krogstrup (2004). For example, the small countries assumption is relaxed by Wildasin (1988), allowing for countries to have an impact on the international after-tax return. As a result, when countries are large, tax rates are still competed downwards but to a lower extent. Similarly, a case of asymmetric countries is considered by Bucovetsky (1991) who shows that larger countries choose higher tax rates. Typically, those additions (large or asymmetric countries) could be implemented in our augmented setting. In this paper we extend the canonical model of Zodrow and Mieszkowski (1986) and keep its core characteristics without adding those kinds of features, although it could be possible. Moreover, the difference between residence and source based taxation is
investigated by Razin and Sadka (1991). Their work shows that residence taxation can if enforceable restore optimum and remove distortions in the provision of the public good.

The second result is that if a second immobile tax base such as labor is used to finance the public good, then the entire tax burned will weight on it. Finally, an important input to the reference framework of international tax competition stems from the literature on new economic geography. Namely, inputs by Andersson and Forslid (2003) and Baldwin and Krugman (2004) show using models of agglomerating economies and differential rents that capital tax rates maintain at a high level in countries with high concentration of production while they decrease in countries with low concentration.

All those features are examples of extensions that build on the seminal framework of Zodrow and Mieszkowski (1986) to amend the key result of the literature that capital mobility results in a downward pressure on tax rates and an under-provision of public goods. Our paper as mentioned in the introduction consists in coming up with a new one that is absent of the current literature: shared federal public goods. We implement in the core model the fact that countries finance together the public good and investigate if its provision is still distorted downwards.

As the literature on tax competition is extensive we can also mention other inputs that enrich the modeling of federalism, such as revenue-sharing, tax deductibility or fiscal equalization. For example, Kelders and Köthenbürger (2005) show that such instruments lead to an upward pressure on local tax rates and to a downward one on federal rates. Again, those features could be added to our model but this would require further work.

Stylized facts on tax competition and downward pressure on tax rates are offered by Devereux et al. (2002). Using data for 16 countries (the EU and G7) from the early 1980’s to late 1990’s, they show, among other stylized facts, that the different tax reforms led to a fall in tax rates and to a broadening in tax bases. From 1982 to 2001, the unweighted mean
statutory tax rate (usual measure for corporate income taxation) decreased from 48 to 35% approximatively, with Ireland having the lowest rate at 10%. The other main finding of the paper is that on average, tax reforms implemented in this group of countries led to a broadening in tax bases. But, as highlighted by the authors of that same paper, seminal papers presented above on tax competition cannot explain such a stylized fact as they do not model tax rates and tax bases separately. And since we build on that canonical part of the literature on tax competition, facts on base broadening is less interesting to us.

A relevant overlap for this paper between the literature on tax competition and the one on fiscal federalism is the concept of vertical externalities and fiscal interdependence. The idea is that the fiscal policy set at one level affects the tax levying capacities at the other level, creating a vertical spillover that can lead to sub-optimal outcomes. That issue has been investigated theoretically by Flowers (1988), Boadway et al. (1998) and Dalhby (2003) which are papers that first studied the importance of vertical externalities in federations. Empirical studies show contradictory results in terms of size and sign of the interaction. Besley and Rosen (1998) find significant evidence of vertical spillover using U.S. data from 1975 to 1989 on gasoline and cigarettes tax rates. Their study shows that a federal tax rate increase triggers a sizable positive response in the local rate. The article of Goodspeed (2000) finds a negative correlation between local and federal rates in 10 OECD countries between 1975 and 1984 using income taxation data. So there exists a sort of competition between local and federal layers, however in our model that interaction is internalized by each country while in the literature those spillover are caused by uncoordinated decision making. Still, tax bases overlapping and being considered as joint property by two levels of government create an inefficiency which can be seen as a negative vertical externality. Those concepts will be well highlighted when introducing the model of overlapping tax bases, in which both levy tax revenue on the same base.
3. Model

3.1 Benchmark model

Our methodology consists in extending a baseline model which is in inspired by Zodrow and Mieszkowski (1986) and synthesized by Krogstrup (2004). That framework introduces a model of tax competition that investigates the impact of capital mobility on tax rates and the provision of public goods. It is a one period model, in which an infinity of identical countries compete for a mobile tax base, each choosing its tax rate as a strategic variable to attract it. Those countries are considered small, which means that the international after-tax return \( r \) is imposed to them without them having any possibility of influencing it. Those countries are composed of three sectors: the private production, a government and one representative citizen. It receives utility from both public and private spending:

\[
\begin{align*}
1) \quad & u(x_i; g_i) \quad u_{x_i}, u_{g_i} > 0 \quad u_{x_i, g_i}, u_{g_i, g_i} < 0 \quad u_{x_i, g_i, g_i} = 0 \\
\end{align*}
\]

The public good \( g \) is financed by taxing capital at the source according to the following budget constraint:

\[
\begin{align*}
2) \quad & g_i = t_i k_i \\
\end{align*}
\]

Hence, capital employed in the country represents tax base for the financing of the public good, and it is also the one and only input to the production process, owned by the representative citizen. Capital has usual decreasing marginal productivity properties:

\[
\begin{align*}
y_i = f_i(k_i) \quad f_i > 0 \quad f_i(k_i) < 0 \\
\end{align*}
\]

The budget constraint of the representative citizen will depend on whether the economy is in autarky or open to the rest of the world, or to put it differently if capital is mobile or not. Given that budget constraint and its own, the government considered as a benevolent entity will choose the tax rate to maximize the utility of the representative citizen.
Under autarky

In the case of autarky, capital cannot flow in and out of the country in response to changes in tax rate: capital mobility is inexistent. As a consequence, the capital employed in the country will be equal to the endowment of that country, \( \bar{k}_i \).

\[ k_i = \bar{k}_i \]

As we are in a one period model, the representative citizen will spend all its income on private consumption, equal to the output minus taxes paid to finance the public good:

\[ x_i = f_i(\bar{k}_i) - t_i \bar{k}_i \]

Utility maximization (1) with respect to the tax rate, subjected to (2) and (3) yields\(^6\) the following first order condition:

\[
\frac{u_g}{u_x} = 1
\]

The government will adjust the tax rate so that the marginal utility of private and public spendings by the representative citizen are equalized. Hence, if capital is immobile, at optimum raising public spendings by one unit means losing exactly one unit of private spendings. As a result, the marginal cost of public funds (MCPF) is one: in autarky, we have a first best optimum in which the public good is efficiently provisioned.

Under open economy

Now that the economy is open to international capital markets, changes in tax rates will cause capital to flow in and out of the country to regain the exogenous after tax return \( r \).

Hence, the following arbitrage condition applies for all \( i \):

---

\(^6\) proof in annex a.

13
An increase in the tax rate will trigger a capital outflow so that the increase in $t$ is met by an equal increase in the gross rate of return of capital to regain the international after-tax return. The extent of that outflow is determined by the elasticity of capital to the tax rate, obtained by totally differentiating the arbitrage condition with respect to the tax rate.

$$f_{k,t} \frac{\partial k_i}{\partial t_i} dt_i - dt_i = 0 \Rightarrow \frac{\partial k_i}{\partial t_i} = \frac{1}{f_{k,t}}$$

which gives us the following elasticity of capital to the tax rate, assumed smaller than one:

$$\varepsilon_k = -\frac{t_i}{k_i f_{k,t}} > 0$$

The budget constraint of the representative citizen now has to include the interests received (paid) as a capital exporter (importer) if the capital employed in the country is lower (higher) that the original endowment.

$$x_i = f_i(k_i) - t_i k_i + r(k_i)$$

Increasing the tax rate now erodes the tax base by causing capital to flow out of the country, leaving less capital to tax. That effect was absent of the autarky case because since capital could not react to a tax increase, the amount of capital to be taxed remaining unchanged, an increase in public spendings resulted in an equal decrease in private spendings. Here, additional to that decrease, a tax base erosion effect occurs and it depends on the elasticity. That difference appears in the maximization of (1) with respect to the tax rate, subjected to (2), (4) and (5)\(^7\):

$$\frac{u_{s_i}}{u_{s_h}} = \frac{1}{1 - \varepsilon_k} > 1$$

\(^7\) proof in annex b.
Hence, the MCPF in open economy is larger that one at optimum: more that one unit of private consumption needs to be sacrificed to increase public consumption by one unit. As such, at optimum, the marginal utility of public spendings is higher than the one of private spending, creating a distortion compared to the autarky case.

In a multi country setting, each country, taking the tax rates of the others as given, has an incentive to decrease its own in order to attract the mobile tax base, leading to race to the bottom. Indeed, each government undercuts the other by setting a lower rate so that capital flows in until the cost in terms of lost tax revenue (due to the tax decrease) and the gain in terms of base inflow balance out. A sub optimal equilibrium is reached, in which the overall amount of capital (across all countries) is the same but the level of public good is inefficiently low. This is the key result of the literature on tax competition: when capital is mobile, public goods set at a sub-optimal level and are under-provided.

**Extending the baseline model with a second public good**

This paper intends to develop a model of tax competition that roots in the one just presented, to which we add a federal layer and a second public good shared at that level. Among the infinity of competing countries, we distinguish two small countries that form a federation and finance a common public good through a federal budget. They still compete to attract an amount of capital that is endowed to the whole federation and mobile between them. Hence we write:

\[ \bar{k}_1 + \bar{k}_2 = \bar{K} = k_1 + k_2 \Rightarrow k_2 = \bar{K} - k_1 \]

The federation remains small: the after tax return is exogenous and imposed to them. The representative citizen now receives utility from three sources: consumption of the private

---

8 the fact that both countries share the total endowment in tax base is inspired by a feature of the seminal paper by Wildasin (1988)
good, of the local public good and of the federal public good. We will use the following utility function:

\[
\begin{align*}
    (6) \quad & u_i(x_i; g_i; G) \quad u_{x_i}, u_{g_i}, u_G > 0 \\
    & u_{x_iG}, u_{g_iG}, u_{GG} < 0 \\
    & u_{x_iG}, u_{g_iG}, u_{gG}, u_{G'}, u_{G'} = 0
\end{align*}
\]

where \( g_i \) is a local good, enjoyed and financed only by country \( i \) and \( G \) is a federal public good shared among the two countries, enjoyed and financed by both.

The objective of the government remains to maximize this utility, using the local and the federal tax rates as strategic variables and subjected to a budget constraint at both those layers. Those constraints will be defined further on, depending on whether we use a model with separate or overlapping tax bases.

### 3.2 Model with different tax bases

We start with a two country model in which the local good is financed by source taxation on physical capital \( k \), and the federal good by source taxation on human capital \( h \). Each country is endowed with an amount of physical and human capital, and the total endowments in the federation will become mobile as countries open. Hence we have:

\[
\begin{align*}
    \bar{k}_1 + \bar{k}_2 = \bar{K} &= k_1 + k_2 \Rightarrow k_2 = \bar{K} - k_1 \\
    \bar{h}_1 + \bar{h}_2 = \bar{H} &= h_1 + h_2 \Rightarrow h_2 = \bar{H} - h_1
\end{align*}
\]

Moreover, as mentioned before, the second good is financed by both countries. The government’s program is subjected to two budgetary constraints, one for each level:

\[
\begin{align*}
    (2) \quad g_i &= t_i k_i \\
    (7) \quad G &= T_1 h_1 + T_2 h_2
\end{align*}
\]

The production function also takes into account the fact that now another type of capital enters the production process with the following properties:
$y_i = f_i(k_i; h_i), \quad f_{k_i} > 0, \quad f_{k_i} < 0, \quad f_{h_i} > 0, \quad f_{h_i} < 0, \quad f_{k_i, h_i} = 0$

**Under autarky**

Both physical and human capital cannot move abroad in reaction of an increase of either local or federal tax rates, respectively. As a consequence, the tax bases will be equal to the initial endowments.

$k_i = \bar{k}_i, \quad h_i = \bar{h}_i$

The budget constraint of the representative citizen is similar to the one in the baseline, but now includes taxes on human capital:

$(8) \quad x_i = f_i(\bar{k}_i; \bar{h}_i) - t_i \bar{k}_i - T_i \bar{h}_i$

Utility maximization of (6) with respect to the local and federal tax rates as strategic variables, subjected to (2), (7) and (8) yields\(^9\) the following system:

\[
\begin{align*}
\frac{u_{G_i}}{u_{x_i}} &= 1 \\
\frac{u_{E_i}}{u_{x_i}} &= 1
\end{align*}
\]

Just as in the baseline model, but now with two layers of government, the marginal costs of public funds (local and federal, respectively MCLPF and MCFPF) are equal to one in the autarky case. That means for the government that an increase in public spendings at one of the two levels (federal or local) translates either into a one to one decrease in private consumption or into a one to one decrease in public spendings at the other level. At optimum, in the autarky case, marginal utilities obtained from the three sources cited above are equalized: both public goods are efficiently provided.

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\(^9\) proof in annex c
**Open economy**

Now that the federation is open to the rest of the world, physical and human capital are accessed internationally through independent and separate markets, their respective after-tax returns being exogenous to the two countries. Changes in the local tax rate will cause physical capital to flow in and out of the country to the other to regain the exogenous after tax return \( r \), just as changes in the federal rate will trigger human capital outflows and inflows to regain \( w \).

Just as before but now with a second market, we have the two following arbitrage conditions:

\[(4) \quad f_k - t_i = r \]
\[(9) \quad f_h - T_i = w \]

An increase in either one of the rates will cause an outflow of the corresponding type of capital so that the gross rate of return meets the international one. How much of that capital has to move abroad depends on the elasticity of that capital to its tax rate. As before, totally differentiating the arbitrage conditions allows to obtain it for both physical and human capital:

\[ \varepsilon_{k_i} = -\frac{t_i}{k_i f_{h_k}} > 0 \]
\[ \varepsilon_{h_i} = -\frac{T_i}{h_i f_{h_h}} > 0 \]

Moreover, the new private budget constraint, comparable to the one in the baseline case under open economy but now with a second type of capital, writes:

\[(10) \quad x_i = f_i(k_i; h_i) - t_i k_i - T_i h_i + r(k_i - k_i) + w(h_i - h_i) \]
The government maximizes the utility function (6) with respect to both tax rates, subjected to the two public budget constraints (2) and (7), the two arbitrage conditions (4) and (9) and the private budget constraint (10) : we obtain the MCPFs\textsuperscript{10} for country $i$.

Considering the local public good and its tax base, physical capital, the baseline situation is as expected replicated. At equilibrium raising local public spendings by one unit means decreasing private spendings by more than one : the marginal cost of local public funds (MCLPF) will necessarily by higher than one, creating a distortion compared to the autarky case.

$$MCLPF_i = \frac{u_k}{u_{k_i}} = \frac{1}{1 - \varepsilon_{k_i}} > 1$$

Hence, the local public good sets at a sub optimally low level.

Analyzing the impact of capital mobility on the federal public good and its tax base, human capital, allows us to obtain the first result of this paper. Contrary to the local good, the federal one is shared which means that is it financed and enjoyed by both countries. As a consequence, if country $i$ raises its federal tax rate, human capital will flow out but it will do so in country $j$ which finances the federal good that country $i$ enjoys as well. At optimum, compared to a situation in which that good is not shared, less private consumption has to be sacrificed to raise federal spendings since the eroding tax base will be taxed in the other country to finance that same federal spending. We have :

$$MCFPF_i = \frac{u_G}{u_{k_i}} = \frac{1}{1 - \left(1 - \frac{T_j}{T_i}\right)\varepsilon_{k_i}} > 1$$

\textsuperscript{10}proof in annex d.
A distortion appears compared to the autarky case but it is lower than the one affecting the local good that is not shared: the marginal cost of federal public funds (MCFPF) is lower\textsuperscript{11}. The ratio reflects an horizontal effect that appears between the two countries at the federal level. It differs from the local good for which the two countries have no way of countering the inefficiency that results from tax competition. They keep undercutting each other to attract physical until equilibrium is reached, delivering a suboptimal outcome. When it comes to the federal public good, they internalize the fact that wherever the tax base is located, it will finance a good that they will enjoy: the need to undercut the other is lower. To put it differently, raising public spendings for a shared public good costs less in terms of private spendings compared to a non shared one.

**Result 1.a.** When capital is mobile, the provision of a shared public good will be less downward distorted than with a non shared public good.

At this point, it is useful to express the equilibrium occurring in the second country\textsuperscript{12}:

\[
\begin{align*}
\text{MCFPF}_1 &= \frac{u_G}{u_{s_1}} = \frac{1}{1 - \left(1 - \frac{T_2}{T_1}\right) \epsilon_h^T} \\
\text{MCFPF}_2 &= \frac{u_G}{u_{s_2}} = \frac{1}{1 - \left(1 - \frac{T_1}{T_2}\right) \epsilon_h^T}
\end{align*}
\]

We observe that the country with the highest federal tax rate has a MCFPF higher than one, while the one with the lowest rate has a MCFPF lower than one, as drawn in figure 1. The country with the highest tax rate has its representative citizen under provided with federal public good while the country with the lowest rate has its over provided with it. From an efficiency point of view, over or under-provision is equally bad, but that means that tax competition with a shared public good does not necessarily leads to a race to the

\textsuperscript{11} assuming strictly positive tax rates, this is always true

\textsuperscript{12} we assume sufficiently low elasticities so that the denominators are strictly positive
bottom. To put it differently, the country with the lowest rate at equilibrium has its representative citizen provided with too large of an amount of federal good for the tax rate that is set in his/her country. Given that rate, it could have had more private good if resources had been allocated differently. Hence, as long as tax rates are different, inefficiencies appear on both ends as a result of uncoordinated behavior across the federation.

**Result 1.b** As long as federal tax rates are different, there will be an over-provision for one country and an under-provision for the other.

**Horizontal equalization**

We observe that if federal tax rates are equal, then the under-provision of one country and the over-provision of the other are wiped out. This can occur if countries are symmetric, in which case federal tax rates align automatically. Or it happens if countries can coordinate at the federal level, provided they have the necessary institutional framework to do so. Such a coordination would allow the federation to depart from the equilibrium reached
under uncoordinated behavior. The importance of fiscal coordination has been recognized by the European Commission (1997), and steps have been taken to harmonize some taxation policies. For example, European countries have been implementing harmonizing reforms regarding VAT (Majocchi, 2008), with a system of investigators to insure compliance to these rules. An horizontal equalization (defined as the alignment of the federal tax rates) yields the equalization of the marginal utility from the consumption of the private good and from the consumption of the federal public good.

Indeed if \( T_1 = T_2 \), then the MCFPF is equal to one just as in autarky in both countries: raising federal spendings translate into a one to one decrease in private consumption.

**Result 1.c** The federation can restore an efficient provision of the federal public good for both countries by setting equal federal tax rates. The local public good remains under provided.

Indeed, at \( T_1 = T_2 \), if country \( i \) raises its federal tax rate assuming country \( j \)’s constant, the lost federal tax revenue corresponding to the fringe of tax base that flows out will be exactly matched by a gain in federal tax revenue in country \( j \) to finance the federal shared good: the horizontal externality is full. But as soon as a country departs from that equilibrium and undercuts the other, inefficiencies immediately appears for both of them as that horizontal effect does not completely apply.

### 3.3 Model with same tax bases

Introducing a second public good, shared at the federal level, results in an horizontal effect that mitigates the inefficiency in the provision of that good. Cooperation even makes it possible to remove the distortion completely. We now turn to a model in which both public goods are financed by taxing physical capital, such as a two-layered corporate income tax.
that would generate both local and federal tax revenues. Hence, human capital disappears completely from the model, going back to the baseline model production function:

\[ y_i = f_i(k_i) \quad f_k > 0 \quad f_{kk} < 0 \]

The new feature of that model is that both public goods have the same tax base. The local one is financed by a local tax and the federal one by a federal tax, both levied on physical capital. Just as before, governments of the two countries will choose their rates to maximize the utility of their representative citizen, under the following public budget constraints:

\[ g_i = t_i k_i \]
\[ G = T_1 k_1 + T_2 k_2 \]

**Under autarky**

Physical capital cannot move abroad when the local or the federal rate increases. As such, the tax base will be equal to the initial endowment in physical capital of the country.

\[ k_i = \bar{k}_i \]

Private consumption follows:

\[ x_i = f(\bar{k}_i) - t_i \bar{k}_i - T_i \bar{k}_i \]

Maximization of (6) subjected to (2), (11) and (12) leads to the same results\(^{13}\) as in the previous model in which tax bases were different. At optimum, marginal utilities from the consumption of the private good, the local public good and the federal public good are equal, implying MCFPF and MCLPF equal to one.

\(^{13}\) proof in annex e.
Increasing the consumption of one of those three by one unit means decreasing one of the other two by one unit (holding the remaining one constant). Having the same MCPF at both layers of government means that both public goods set the same level. And since they have the same tax base, the two tax rates are equal as well.

\[
\begin{align*}
\frac{u_{s_i}}{u_{x_i}} = 1 \\
\frac{u_{G_i}}{u_{x_i}} = 1
\end{align*}
\]

**Under open economy**

Capital can now flow out towards the other country in reaction of an increased tax rate at either one of the level of government (local or federal). Previous assumptions on the nature of the federation (small countries) still apply, hence we have the following arbitrage condition:

\begin{equation}
(13) \quad f_i - t_i - T_i = r
\end{equation}

An increase in either one of the rates will trigger a capital outflow to regain the world after-tax return. The size of that outflow depends on the elasticity of physical capital to that tax rate. Totally differentiating the arbitrage conditions allows to obtain those two elasticities (one for the local tax rate, one for the federal one):

\[
\varepsilon_{k_i}^l = -\frac{t_i}{k_i f_{k_i}} > 0
\]

\[
\varepsilon_{k_i}^f = -\frac{T_i}{k_i f_{k_i}} > 0
\]
Private budget constraint in this framework includes input minus tax paid at the local and the federal levels, plus (minus) interests received (paid):

\[
x_t = f_i(k_i) - t_i k_i - T_i k_i + r(k_i - k_f)
\]

We saw in the previous section that sharing the federal public good, while financing it with a different tax base than the local public good, allows to trigger horizontal effects that mitigate the inefficiency in its provision. The non shared local good remains under provided, as no such effects can be exploited.

Now, each government when choosing its rates takes into account that, first, the financing of the federal good is shared and, second, the fact that both goods are financed through taxation of physical capital. Those interactions produce new substitution effects that change the equilibrium. Indeed, now that we have one tax base for two public goods, capital mobility produces vertical (between the local and federal levels of government of the same country) and cross effects (between different level of government of different countries) on top of the horizontal ones. Those interactions relate to the externalities mentioned in the fiscal federalism literature.

In our model, an increased federal rate will cause capital to flow out, leading to a shrinking in the base of the federal good, but in the base of the local good as well. So even if the local rate is held constant, the level of the local public good will decrease as there is less capital to tax to finance it. The same goes for the local rate towards the provision of the federal good. Having different bases (physical and human capital) means that the equilibrium taking place at one level of government has no impact on the one happening at the other level. Now that both public goods are financed through taxation of the same base, raising the rate of one of them creates a negative externality for the other: this is the vertical effect. It is a sign of vertical spillover existing between the two layers of the
government, something that is well highlighted in the literature on overlapping tax bases as mentioned in section 2.

As the federal good remains shared, the horizontal effects that already appeared in the previous section still arise: raising the federal tax rate makes capital flow out, but in the other country that finances that same good, mitigating its under-provision.

Finally, the overlapping tax bases now link the federal layer of each country to the local one of the other, and vice versa. Indeed, raising the federal rate will cause some tax base to flow in the other country that will be able to finance the federal good but also more of its own local good, which is not shared. Hence, some of the eroding tax base in country \( i \) will be « diverted » to the financing of the local good of country \( j \), while in the case of different tax bases this was impossible. We call this externality a cross effect that, as explained further, will lead to the under-provision of the federal public good under horizontal equalization.

Totally differentiating the arbitrage condition with respect to both tax rates in combination with the definitions of the elasticities allows us to solve further and to obtain the following result\(^{14} \) at equilibrium:

\[
u_g = u_G
\]

This means that at equilibrium, with the local and the federal public goods having overlapping tax bases, the local and federal tax rates are equalized (which we call vertical equalization, we note \( \tau \) that rate that hence applies at the local and the federal level) leading to the same level of local and federal public goods, and hence the same marginal utility. We note that this result is consistent with the fact that we had equal local and federal tax rates at optimum in autarky as well.

\(^{14}\) proof in annex f
**Result 2.a** When two public goods have the same tax base, at equilibrium the tax rates that finance them are equal. Hence their provision is equal.

Indeed, the government of each country has the capacity of internalizing the vertical effects by setting equal tax rates, canceling out the relative distortion between the provision of the local and the federal good.

Since we have the same tax rates, elasticity of capital with respect to both rates is the same:

\[ \varepsilon^*_k = -\frac{\tau_i}{k_i f_{kk}} \]

Those results allow us to express\(^{15}\) the MCLPF and MCFPF\(^{16}\):

\[
MCLPF_i = MCFPF_i = \frac{1}{1 - \left(2 - \frac{\tau_i}{\tau_i}\right) \varepsilon^*_k}
\]

As the level of the two public goods and their corresponding tax rate are the same, is it clear that they will be equally provided, and that raising the level of one or the other will cost the same amount in terms of private consumption. As such, MCLPF and MCFPF are the same, we will refer further to the MCPF without distinction between the local or the federal level. To analyze those results further we can express the equilibrium happening in both countries:

---

\(^{15}\) we use the equality of marginal utilities from the public goods and plug it into either one of the first order conditions

\(^{16}\) again assuming low enough elasticities so that we have strictly positive denominators
Depending on how capital is allocated across the federation given the arbitrage conditions and elasticities of the two countries, equilibrium can happen according to four different cases, represented in figure 2:

\[
\begin{align*}
\text{MCPF}_i &= \frac{u_G}{u_{x_i}} = \frac{u_G}{u_{x_j}} = \frac{1}{1 - \left(2 - \frac{\tau_2}{\tau_1}\right)\epsilon_{k_i}^r} \\
\text{MCPF}_j &= \frac{u_G}{u_{x_i}} = \frac{u_G}{u_{x_j}} = \frac{1}{1 - \left(2 - \frac{\tau_1}{\tau_2}\right)\epsilon_{k_j}^r}
\end{align*}
\]

Case a. One country has over-provided public goods, the other has its under-provided if \(2\tau_i < \tau_j\). If at equilibrium a country sets its rates more than twice as high as the other, then its public goods are under-provided while they are over-provided in the other country. Indeed, if the tax rates of country \(j\) are that high due to uncoordinated behavior, then a lot of the tax base locates in country \(i\) and serves as base for its local good as well, which is then over-provided considering its own lower tax rate. Hence, it is possible to have an over-provision of the local public good, while the under-provision was unavoidable in previous models. Conversely, setting rates that high means for country \(j\) having a lot of capital outflow towards country \(i\) that doesn’t provide that much federal
tax revenue to the federal public good: as a result it is under provided for country $j$. That downward distortion due to the fleeing tax base impacts on the provision of the local good, making it under provided as well.

**Case b.** The public goods are under provided in both countries if $2\tau_i > \tau_j$. As long as no country has its tax rates more than twice as high as the other, both of them has its representative citizen under provided with the local and the federal public goods. Moreover, the country that has the highest rate has the strongest downward distortion due to base erosion.

**Case c.** One country has an efficient provision of its public goods while the other has its under provided if $2\tau_i = \tau_j$. At this equilibrium, we have the following results:

$$\begin{align*}
&MCPF_i = 1 \\
&MCPF_j = \frac{1}{1 - \frac{3}{2} \epsilon_{i}'}
\end{align*}$$
For country $i$, optimum is fully restored at the local and the federal levels just as in the autarky case. The MCFPF and the MCLPF are both equal to one, hence raising public spendings at a level of government by one unit translates into a one to one decrease in private consumption or in public spendings at the other level, creating no inefficiency. Hence, for those rates, the allocation of capital across the federation settles so that optimum happens in country $i$. However, country $j$ has its own public goods necessarily under provided. In the model with different tax bases, the federation was able to obtain an efficient provision of the federal public good for both countries (and could not do anything about the under-provision of the local public good) by setting equal federal tax rates. Here, it can reach an efficient provision for both public goods but for only one country at once. Having MCPF equal to one for the two countries at the same time is now impossible.

**Result 2.b** With overlapping tax bases, the federation can restore full autarky optimum but only for one of the two countries at once.

This equilibrium is hardly tenable politically. Country $j$ would have to accept having that high of a tax rate to restore optimum in country $i$ at the expense of the utility of its own representative citizen.

**Case d. Both countries have equal under-provision of their public goods:** horizontal equalization happens. In the previous model, horizontal equalization (alignment of the two federal tax rates) that stemmed either from symmetry or cooperation allowed to restore the efficient provision of the federal public good for both countries. As the MCFPF was equal to one, it was neither under nor over-provided. Here, if the two countries of the federation equalize their federal tax rates, they will have the
same provision of federal and local public goods but it will not be efficient anymore. We have the following ratios after setting equal tax rates for both countries$^{17}$:

$$MCPF_i = MCPF_j = \frac{1}{1 - \epsilon_k^i}$$

First, we note that we go back to the same MCLPF as in previous models. Hence, the local public good of each country sets at the same sub-optimal level as in the baseline model and the model with different tax bases: raising its level will cost the same amount in terms of private consumption. Second, base erosion affects both levels of government but the one happening at the federal level will be cancelled out because of the horizontal equalization. To put it differently, at $\tau_1 = \tau_2$, country $i$ knows that if it raises its rates assuming country $j$'s ones constant, the loss of federal tax revenue due to the capital outflow will be exactly compensated by a gain in federal tax revenue in country $j$. This allowed the efficient provision of the federal public good in the previous model. Here however, if countries do not compete anymore for capital at the federal level, they still compete for that same tax base at the local level to finance their respective local public good. That competition is of the same nature than the one happening in the baseline model in which there was only a single non shared public good, yielding that sub-optimal equilibrium. And now that bases are linked, country $i$ knows that raising its tax rates will cause tax base to flow out, generating tax revenue for country $j$'s local good as well which country $i$ does not enjoy. Hence, tax competition at the local level is replicated on the federal level: this is the cross effect that sets the federal good at an inefficiently low level compared to the model with different tax bases. As a consequence, the provision of the federal good is equally downward distorted: the MCFPF is necessarily higher than one.

$^{17}$ again, either with symmetry or cooperation
Comparing equilibria under horizontal equalization

Let us compare the equilibria of the two models (with different and with same tax bases) when the two countries harmonize their tax rates (either because they have the institutional framework to cooperate or because they are symmetric). If tax bases are different, we have the following ratios:

\[
\begin{align*}
MCLPF_i & = \frac{u_k}{u_x} = \frac{1}{1 - \epsilon_k} > 1 \\
MCFPF_i & = \frac{u_G}{u_x} = 1
\end{align*}
\]

If tax bases are the same, we have those:

\[
\begin{align*}
MCLPF_i & = \frac{u_k}{u_x} = \frac{1}{1 - \epsilon_k} > 1 \\
MCFPF_i & = \frac{u_G}{u_x} = \frac{1}{1 - \epsilon_k} > 1
\end{align*}
\]

We can see that the local good is necessarily under-provided, to the same extent in both cases. However, in the model with different tax bases, only the provision of the local public good is downward distorted. In the model with a single tax base, both public goods will be equally under provided.

**Result 2.c** Under horizontal equalization, overlapping tax bases replicate the inefficiency affecting the local good onto the provision of the federal good, distorting it to the same extent. With different tax bases, this was absent.

Comparing those two equilibria yields the well known result of the literature on overlapping tax bases. With separate bases, tax rate harmonization leads to the efficient provision of the federal public good. With the two layers of government sharing the same tax base, that same harmonization produces an inefficient provision and hence inefficient
decision making. This is a cause of concern for federations that intend to harmonize their tax rates at the federal level if those rates levy revenue on bases that overlap with taxes at the local level.

3 Extension: a model with transfers

We developed in the previous section a model of tax competition with concepts of shared public goods. Using those, we move to a model with different tax bases in which the federal public good of country 1 is only partially shared. Indeed, we set up a model in which country 2 transfers part of the provision of its federal public good to country 1, meaning that country 2 finances part of its own federal good and part of country 1’s. Hence we have the following utility function:

\[ u_i(x_i; g_i; G_i) \]

with the three sources of utility having the same properties as before.

The extent of the transfers is measured with the parameter \( \theta \), according to the following updated federal budget constraints:

\[ G_1 = T_1h_1 + \theta T_2h_2 \]
\[ G_2 = (1 - \theta)T_2h_2 \]

where \( \theta < 1 \). As such, the higher the \( \theta \), the higher country 2 transfers federal spendings to the other country of the federation. As \( \theta \) tends to zero, we go back to a situation in which the public goods are not shared and the distortion due to tax competition is at its highest. Indeed, we get back to the usual budget constraints with non shared goods. As it tends to one, country 2 forfeits utility from its federal good and transfers its whole federal tax revenue to finance country 1’s, as shown below.
The rest of the model follows the equations of the previous model with different tax bases, in terms of local public budget constraint (2), private budget constraint (10) and arbitrage conditions (4) and (9). Maximization by the governments with respect to the local and federal tax rates now yields\(^\text{18}\) the following MCFPF for the federation:

\[
MCFPF_1 = \frac{1}{1 - \left(1 - \theta \frac{T_2}{T_1}\right) \epsilon_h^T_i} \\
MCFPF_2 = \frac{1}{(1 - \theta)(1 - \epsilon_h^T_i)}
\]

Let us remind that, without transfers (if theta were equal to zero), we would have the following ratios:

\[
\begin{cases}
MCFPF_1 = \frac{1}{1 - \epsilon_h^T_i} \\
MCFPF_2 = \frac{1}{1 - \epsilon_h^T_i}
\end{cases}
\]

which are the ones we get with non shared goods.

Comparing those results shows that the MCFPF for country 2 is necessarily higher with transfers, and increasing in \(\theta\) : now that there are transfers, raising the federal rate means forfeiting part of the corresponding tax revenue to country 1, which implies an additional cost on top of losing tax base. That cost is reflected in the ratio of MCFPF for country 2, and it leads to a more pronounced under-provision of federal good for that country.

Moreover, observing MCFPF for country 1, which benefits from the transfers, shows it is less costly in terms of private consumption to raise its federal spendings. This is due to an

\(^{18}\) proof in annex g. Moreover, here we only consider the case of open economy, as it doesn’t make much sense to study autarky in a model with transfers. Plus, we only display results for MCFPF as the local level of the problem remains unchanged.
interaction of the type of the horizontal externality developed in previous section, although
reduced. Indeed, country 1 has lowered incentives to undercut the other country to attract
its tax base as that base finances its own federal good as well. As a result, the MCFPF for
country 1 is necessarily reduced as long as $\theta$ is positive and transfers occur. In fact, the
MCFPF is decreasing in $\theta$ as more transfers translate into a less costly base erosion effect.

**Result 3.a** When a country of the federation transfers part of its federal tax revenues
to the other, its MCFPF is increased while the one of the receiving country is reduced.

Note that we can also compare those MCFPF to the ones obtained in the regular model of
different tax bases, in which the federal good was fully shared. In that model, both
countries finance the federal good and both enjoy it. Here, for the country that receives the
federal transfers, the MCFPF is lower with transfers compared to a situation in which the
good is not be shared, but higher than in the model with different tax bases:

\[
1 - \left(1 - \frac{T_1}{T_1} \right) \varepsilon_{h_1}^T < 1 - \left(1 - \theta \frac{T_2}{T_1} \right) \varepsilon_{h_1}^T < \frac{1}{1 - \varepsilon_{h_1}^T}
\]

We see that transfers are less useful of a fiscal institution to counter the under-provision of
the public good for country 1 than a fully shared public good. Indeed, an outflow of human
capital due to a federal rate increase translates into less of a horizontal externality than
with a fully shared public good. Still, transfers help to mitigate the under-provision of that
good compared to a situation in which the good is not shared.

Besides, horizontal equalization ($T_1 = T_2$) doesn’t restore optimum anymore. Here, setting
equal tax rates leads to the following MCFPF:
which is necessarily higher than one.

**Result 3.b** When a country of the federation transfers part of its federal tax revenues to the other, optimum cannot be restored by harmonizing tax rates anymore.

Getting MCFPF for country 1 closer to one would require from fiscal authorities to set $\theta$ closer to one as well, which increases MCFPF for country 2. Hence in a model of transfers the federation has to sacrifice utility in country 2 in order to remove inefficiencies in country 1, which raises political feasibility issues.

5 Conclusion

Our paper showed that the classic conclusions of the literature on tax competition can be put in perspective when we consider the case of a federation of countries that finance together a public good, relating with the literature on fiscal federalism. Those models introduce the fact that in the case of shared federal public goods, the base that erodes goes to a country that finances the good enjoyed by the very country that lost some of its base. In both models, those interactions can, depending on the case, mitigate the under-provision of public goods or even lead to an over-provision.

In a model with different tax bases, the federation can restore an efficient provision of the federal good for both countries at the same time by harmonizing tax rates. While in a model with single tax base, this is made impossible as only one country can enjoy an efficient provision of the federal good at once, however optimum applies for the local good
as well. In that model, harmonizing tax rates replicates the inefficiency to the federal level.

We also proposed a model with transfers, in which the MCFPF of a country can be reduced at the expense of the other country. However, optimum is impossible to reach.

Potential extensions to that framework include the ones presented in the literature review.

We can also imagine that trade in private good between the countries could happen and trigger additional interactions.

Another potential extension could be to extent the federation by include more countries and see if the results presented above still hold. This would be a relevant analysis if we consider the case of the European Union that keeps including more member countries while attempting to harmonize tax rates.
Bibliography


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Annex

**Proof a. Autarky equilibrium in the baseline model**

\[
\max_{t_i} u_i(f_i(k_i) - t_i k_i; t_i k_i)
\]

First order condition writes
\[
\frac{\partial u_i}{\partial t_i} = -k_i u_{x_i} + \bar{k}_i u_{x_i} = 0
\]

\[
\Rightarrow \frac{u_{x_i}}{u_{x_i}} = 1
\]

**Proof b. Open economy equilibrium in the baseline model**

We can rewrite the private budget constraint (5) and use the arbitrage condition (4):

\[
x_i = f_i(k_i) - (t_i + r) k_i + r(k_i)
\]

Hence we have the following maximization program:

\[
\max_{t_i} u_i(f_i(k_i) - f_i k_i + r k_i; t_i k_i)
\]

First order condition writes
\[
\frac{\partial u_i}{\partial t_i} = \left(f_i \frac{\partial k_i}{\partial t_i} - f_i h_i \frac{\partial k_i}{\partial t_i} - f_i k_i \frac{\partial k_i}{\partial t_i} \right) u_{x_i} + \left( k_i + t_i \frac{\partial k_i}{\partial t_i} \right) u_{x_i} = 0
\]

\[
\Rightarrow \frac{u_{x_i}}{u_{x_i}} = \frac{f_i k_i \frac{\partial k_i}{\partial t_i}}{k_i + t_i \frac{\partial k_i}{\partial t_i}} = \frac{1}{1 - \varepsilon_{h_i}}
\]

**Proof c. Autarky equilibrium in the model with different tax bases**

\[
\max_{t_i; h_i} u_i(f_i(k_i; h_i) - t_i k_i - T_i h_i; t_i k_i; T_i h_i + T_2 h_i)
\]

First order conditions write
\[
\frac{\partial u_i}{\partial t_i} = -k_i u_{x_i} + \bar{k}_i u_{x_i} = 0
\]

and
\[
\frac{\partial u_i}{\partial T_i} = -h_i u_{x_i} + \bar{h}_i u_{G} = 0
\]

\[
\begin{cases}
  u_{x_i} = 1 \\
  u_{x_i} \\
  u_{G} = 1 \\
  u_{x_i} = 1
\end{cases}
\]
Proof d. Open economy equilibrium in the model with different tax bases

We can rewrite the private budget constraint (10) and use the arbitrage conditions (4) and (7):

\[ x_i = f_i(k_i;h_i) - (t_i + r)k_i - (T_i + w)h_i + r(k_i) + w(h_i) = f_i(k_i;h_i) - f_i(k_i) - f_i(h_i) + r(k_i) + w(h_i) \]

Hence we have the following maximization program:

\[ \text{Max} \quad u_i \left( f_i(k_i;h_i) - f_i(k_i) - f_i(h_i) + r(k_i) + w(h_i); t_i k_i; T_i h_i + T_j \left( H_i - h_i \right) \right) \]

First order conditions write

\[ \frac{\partial u_i}{\partial t_i} = \left( f_i \frac{\partial k_i}{\partial t_i} - f_i \frac{\partial k_i}{\partial t_i} + f_i \frac{\partial k_i}{\partial t_i} \right) u_t + \left( k_i + t_i \frac{\partial k_i}{\partial t_i} \right) u_t = 0 \]

and

\[ \frac{\partial u_i}{\partial T_i} = \left( f_i \frac{\partial h_i}{\partial T_i} - f_i \frac{\partial h_i}{\partial T_i} - f_i \frac{\partial h_i}{\partial T_i} \right) u_t + \left( h_i + T_i \frac{\partial h_i}{\partial T_i} - T_i \frac{\partial h_i}{\partial T_i} \right) u_t = 0 \]

\[ \Rightarrow \begin{cases} 
  u_{h_i} = f_{h,k} \frac{\partial k_i}{\partial t_i} = \frac{1}{1 - \epsilon_h} \\
  u_{k_i} = k_i + t_i \frac{\partial k_i}{\partial t_i} = \frac{1}{1 - \epsilon_h} \\
  u_{G_i} = f_{h,h} \frac{\partial h_i}{\partial T_i} - h_i + T_i \frac{\partial h_i}{\partial T_i} - T_i \frac{\partial h_i}{\partial T_i} = 1 - \left( 1 - \frac{T_i}{T_i} \right) \epsilon_h 
\end{cases} \]

Proof e. Autarky equilibrium in the model with same tax bases

\[ \text{Max} \quad u_i \left( f_i(k_i) - t_i k_i - T_i k_i; t_i k_i; T_i k_i + T_j k_j \right) \]

First order conditions write

\[ \frac{\partial u_i}{\partial t_i} = -k_i u_i + k_i u_i = 0 \]

and

\[ \frac{\partial u_i}{\partial T_i} = -k_i u_i + k_i u_i = 0 \]

\[ \Rightarrow \begin{cases} 
  u_{h_i} = 1 \\
  u_{k_i} = 1 \\
  u_{G_i} = 1 \\
  u_{l_i} = 1 
\end{cases} \]
Proof f. Open economy equilibrium in the model with same tax bases

We can rewrite the private budget constraint (14) and use the arbitrage condition (13):

\[ x_i = f_i(k_i) - (t_i + T_i + r)k_i + r(k_i) + f_k k_i + r 

Hence we have the following maximization program:

\[ \max_{t_i, T_i} u_i \left( f_i(k_i) - f_k k_i + r k_i; t_i, T_i; k_i + T_1 (K - k_i) \right) \]

First order conditions write

\[ \frac{\partial u_i}{\partial t_i} = \left( f_i \frac{\partial k_i}{\partial t_i} - f_k k_i \frac{\partial k_i}{\partial t_i} \right) u_i + \left( k_i + t_i \frac{\partial k_i}{\partial t_i} \right) u_i + \left( T_i \frac{\partial k_i}{\partial t_i} - T_{ij} \frac{\partial k_i}{\partial T_j} \right) u_i = 0 \]

and

\[ \frac{\partial u_i}{\partial T_i} = \left( f_i \frac{\partial k_i}{\partial T_i} - f_k k_i \frac{\partial k_i}{\partial T_i} \right) u_i + \left( t_i \frac{\partial k_i}{\partial T_i} \right) u_i + \left( k_i + T_i \frac{\partial k_i}{\partial T_i} - T_{ij} \frac{\partial k_i}{\partial T_j} \right) u_i = 0 \]

Totally differentiating the arbitrage condition with respect to both tax rates yields:

\[ f_k \frac{\partial k_i}{\partial t_i} dt_i - dt_i = 0 \]

and

\[ f_k \frac{\partial k_i}{\partial T_i} dT_i - dT_i = 0 \]

\[ \frac{\partial k_i}{\partial t_i} = \frac{\partial k_i}{\partial T_i} = \frac{1}{f_k} \]

Using this result and dividing the order conditions by \( k_i \):

\[ \Rightarrow u_i = u_G \]
Proof g. Equilibrium in a model with transfers

We can rewrite the private budget constraint (10) and use the arbitrage conditions (4) and (7):

\[ x_i = f_i(k; h) - (t_i + r)k_i - (T_i + w)h_i + r\bar{k} + w\bar{h} = f_i(k; h) - f_i k_i - f_i h_i + r\bar{k} + w\bar{h} \]

Hence we have the following maximization program for country 1:

Max \( u_1 \left( (f_i(k; h) - f_i k_i - f_i h_i + r\bar{k} + w\bar{h}; t_i; T_i; H) \right) \)

First order condition with respect to the federal tax rates writes

\[
\frac{\partial u_1}{\partial T_1} = \left( f_{h/k} \frac{\partial h_1}{\partial T_1} - f_{h/k} \frac{\partial h_1}{\partial T_1} - f_{h/k} \frac{\partial h_1}{\partial T_1} \right) u_n + \left( h_1 + T_1 \frac{\partial h_1}{\partial T_1} - \theta T_2 \frac{\partial h_1}{\partial T_1} \right) u_G_1 = 0
\]

\[
\Rightarrow \frac{u_G_1}{u_n} = \frac{f_{h/k} \frac{\partial h_1}{\partial T_1}}{h_1 + T_1 \frac{\partial h_1}{\partial T_1} - \theta T_2 \frac{\partial h_1}{\partial T_1} = 1 - \left( 1 - \frac{T_2}{T_1} \right) \epsilon_{k_1}}
\]

We have the following maximization program for country 2:

Max \( u_2 \left( (f_2(k; h) - f_2 k_2 - f_2 h_2 + r\bar{k}_2 + w\bar{h}_2; t_2, k_2, T_2, T_2, (1-\theta)T_2h_2) \right) \)

First order conditions with respect to the federal tax rates writes

\[
\frac{\partial u_2}{\partial T_2} = \left( f_{h/k} \frac{\partial h_2}{\partial T_2} - f_{h/k} \frac{\partial h_2}{\partial T_2} - f_{h/k} \frac{\partial h_2}{\partial T_2} \right) u_n + (1-\theta) \left( h_2 + T_2 \frac{\partial h_2}{\partial T_2} \right) u_G_2 = 0
\]

\[
\Rightarrow \frac{u_G_2}{u_n} = \frac{f_{h/k} \frac{\partial h_2}{\partial T_2}}{(1-\theta) \left( h_2 + T_2 \frac{\partial h_2}{\partial T_2} \right) = 1 - \left( 1 - \theta \right) \left( 1 - \epsilon_{k_2} \right)}
\]