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GRENOBLE iae Graduate school of Management

Research Thesis

Excess Volatility in Equity Risk Premium Model

Presented by: HACHFI Chaimaa

University advisor: GIRERD-POTIN Isabelle

Master 2 Research
Program Advances in Finance and Accounting
2016 - 2017

UNIVERSITÉ Grenoble Alpes

Mémoire de recherche

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Preface:

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HACHFI Chaimaa

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15/06/2017

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After several days of work and hard work, the results and fruits are well presented in the present dissertation, but in reality the work started largely before these days. It was such an honor and such a good decision to work hardly for giving the best of my work within this master program dissertation.

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AVANT-PROPOS

In asset market, investors put their money in the market to realize profit, they buy assets then sell them to get capital gains and make their business growing more and more. But to make profit, they must well trade assets in the market, buy them when they are underpriced and sell them when they are overpriced, nevertheless, the ordinary strategies of trading are known by all agents in the market, and they are unreliable to gain money from trading, if they don't make investors losing money because of the inverse reaction of the market when all investors undertake the same strategies. From all these reasons, it seems how difficult is trading in stock market and how much awareness it requires to make profit in highly competitive market. Thus, in a dynamic market, investors are supposed to make relevant and reliable expectations to take decisions consistent with market movements, but when competitiveness become high, prices may appear unforecastable. Asset pricing models have been conceived to model the dynamic of the market and explain how prices are moving with respect to a set of factors, a lot of researchers attempted to propose models representing the real market as closer as possible. Since, many asset pricing models have been suggested and discussed regarding their validity and their efficiency for several decades.

In stock market, investors rely intensively on stocks over-valuations and under-valuations to expect future movements in stock prices, selling stocks whose price will decrease and buying stocks whose price will increase. But what if these stocks are too volatile and incur several sequences of over-valuations and under-valuations in short period; surely investors' expectations will be less accurate and more uncertain. Recently, in 1981, Robert Shiller talked in his paper about the difference between prices provided by efficient markets model and market prices; he illustrated this difference using a new term "Excess volatility" as the excess dispersion of market prices relatively to efficient market prices. Since, this new concept has pre-occupied very important part in asset pricing models; several researchers have been interested in exploring and explaining excess volatility in stocks prices.

ABSTRACT

In stock market, investors are looking for profit, they buy stocks and sell others, but before buying or selling stocks they must think carefully because each decision may be the path to losses. Investors are concerned by expecting future movements in stock prices to sell stocks whose process will decrease and buy stocks whose prices will increase. Their expectations must be built in basis of stocks under-valuations and over-valuations. But, in 1981 Shiller introduced new concept in stock market which is excess volatility. Excess volatility is the excess dispersion of stock prices relatively to their fundamental values. This new concept implies that stocks are too volatile and they incur several sequences of under-valuations and over-valuations in short period, hence investors' expectations will be less accurate and more uncertain. This involves investors to be aware of excess volatility risk. In this study, we reported that several researchers proved the imminent existence of stock prices excess volatility in stock market. The best way to get aware of excess volatility risk is to understand the factors behind excess volatility**.** Indeed, four factors are the sources of excess volatility in stock prices, Knightian uncertainty, learning, investors' behavior irrationality and overconfidence. Excess volatility is a common risk factor, this property make it one of common risk factors that must be integrated in equity risk premium model. But this step requires eventually a previous one; which is to find a proxy to stock prices excess volatility. As market stock prices are mean reverting towards their fundamental values, this specification was the first basis of excess volatility proxy we proposed in this study.

Key Words: Excess volatility – fundamental value – Mean reversion – Equity risk premium - Modelling

RESUME

Dans le marché des actifs, l'objectif principal des investisseurs est de réaliser des profits, ils achètent des actifs et vendent des autres, mais avant d'acheter ou de vendre, ils doivent bien réfléchir car toute décision prise peut être le chemin vert des immenses pertes. Les investisseurs sont concernés par prévoir les futurs mouvements dans les prix des actifs pour vendre les actifs dont le prix va décroitre et acheter les actifs dont le prix va accroitre. Leurs prévisions sont basées sur les sous-évaluations et les sur-évaluations des prix des actifs. Mais en 1981, Robert Shiller a introduit un nouveau concept au marché des actifs qui est l'excès de la volatilité. L'excès de la volatilité est l'excès de dispersion des prix des actifs relativement aux valeurs fondamentales de ces actifs. Ce nouveau concept implique que les actifs sont trop volatile et ils subissent des multiples séquences des sur-évaluations et des sous-évaluations dans une période courte, alors les prévisions des investisseurs vont être moins précises et plus incertaines. Ceci implique les investisseurs d'être plus vigilant à propos de risque de l'excès de la volatilité. Dans cette étude, nous avons reporté que plusieurs chercheurs ont approuvé l'existence éminente de l'excès de volatilité dans les prix des actifs. La meilleure façon pour être attentif du risque de l'excès de la volatilité c'est de comprendre les facteurs derrière. Effectivement, quatre facteurs sont à la base des sources de l'excès de la volatilité dans les prix des actifs, Knightian uncertainty, learning, investors' behavior irrationality and overconfidence. L'excès de la volatilité est un facteur de risque commun, cette propriété lui rendre parmi les facteurs de risque communs qu'ils doivent être intégrés dans le model de prime de risque des actifs. Mais cette étape exige bien une étape antérieure ; elle s'agissait de trouver un proxy à l'excès de volatilité des prix des actifs. Comme les prix de marché des actifs ont un processus de retour à la moyenne vers leurs valeurs fondamentales, cette spécificité était la première base du proxy que nous avons associé à l'excès de volatilité dans notre étude.

MOTS CLÉS : L'excès de volatilité – valeur fondamental – Retour à la moyenne – Prime de risque– Modélisation

INTRODUCTION

Excess volatility has been introduced the first time by Robert Shiller in 1981. He defined it as the excess dispersion of market prices that cannot be explained by the volatility of dividends. As stock price is unknown, it is expected by agents as the discount value of future dividends $P_t^* = \sum_{k=0}^{\infty} \frac{d_{t+k}}{(1+R)^{1+k}}$.

As future dividends are not known at time t except the next one; P_t^* will then be characterized as random variable with mean $E_t(\ P_t^*)$ and $Var(\ P_t^*\ )$. We define the expected price in the market as follows: $P_t = E_t(\ P_t^*)$. Since P^* is the true value and P is the expectation of this value, we may expect some differences between P and P^*, but these differences must be slight and not noticeable. Nevertheless, the data of market price does not fit conventional dividends asset pricing model; Dividends asset price P_t^* is much less volatile than market price P_t. Robert Shiller used and still uses in his conferences the same graph showing the difference between Dividends stock price P_t^* and market stock price P_t. He used historical data to present both market prices and theoretical prices computed through dividends model.

Figure 1 : Graphical Representation of excess volatility: Dividends price P^* and market price P

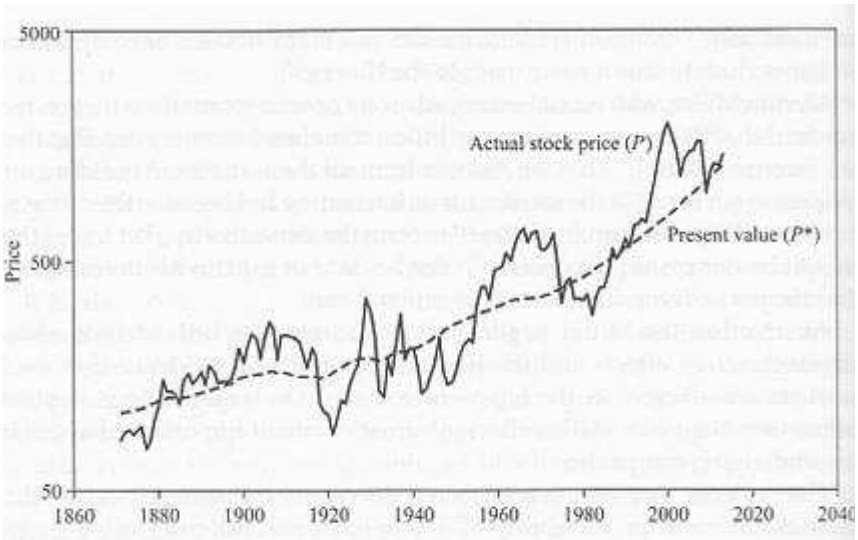

Figure 11.2
Stock Prices and Dividend Present Values, 1871–2013

This figure is extracted from Irrational Exuberance book written by Robert Shiller 2013

As we see in the graph historical prices computed using the present value of dividends are presented as smooth trend whereas market prices' series are too volatile to be explained by dividends volatility. The excess volatility as shows the graph above is the volatility which is

unpredictable, it is explored as the difference between realized prices volatility and dividends expected prices volatility. Excess volatility can be result of irrational as well rational volatility in investors' behavior and how much their decisions become less or more aggressive. It expresses high frequency of over-increases and over-decreases in market prices; hence some investors may build highly profitable trading strategies based on these over movements whereas there would be others undergoing big losses because of stock prices excess volatility. Since it might create either losses or gains, it requires lending huge importance for both research as it is subsequent to previous researches in asset pricing models and in asset management because it makes managers aware of its effects, its sources as well as how make to cover its risk. We chose to deal in this thesis with Excess volatility factor and how it affects Equity Risk Premium. In the first section we reported a brief review of findings in the previous research papers. Then, in the second section, we explore the concept of excess volatility and volatility bounds tests, in the third section, we shed the light on main factors that are responsible for excess volatility, in the fourth section, we explain the relation between excess volatility and the equity risk premium in a way to suggest a new asset pricing model when excess volatility factor takes place besides the other factors.

CHAPTER 1 – LITERATURE REVIEW

Excess volatility has pre-occupied very important part in asset pricing studies, since it illustrates one of the most consistent anomalies of asset pricing models. It consists on the excess dispersion of asset market prices relatively to asset prices given through efficient market models. The first exploration of excess volatility was suggested by Robert Shiller in 1981, in his research paper, he noticed that the movements in stock prices are too big relatively to actual movements in both dividends and nominal stock prices. He then reported that this excess volatility of market prices is the volatility quantity which cannot be explained by dividends volatility and it is due to other factors generating high dispersion in market stock prices. Robert Shiller was the first to talk about Excess Volatility; he discussed in his research paper the limits on stock price volatility imposed by asset pricing models, he established three inequalities that are deduced from theoretical asset pricing models. In basis of asset pricing models the volatility of theoretical stock prices is sum of volatility of market stock prices and disturbance term which represents the set of noises, ambiguities and missed expectations. Thanks to this equality, Robert Shiller found three inequalities that all of them are equivalent to the fact that the volatility of theoretical stock prices is greater than the volatility of market stock prices. Contrary to these three inequalities, data from the market shows absolutely the opposite; the volatility of theoretical stock prices that must exceed the one of market stock prices is too much exceeded by this latter. This anomaly or this contradiction between theoretical facts and real data from stock market gave birth to the new concept in research; and from this instant researchers became curious about the facts surrounding this anomaly or what they called stock prices excess volatility. The data used by Robert Shiller in his first research article shedding the light on excess volatility, exhibits violations of the three inequalities and proves that in reality, the volatility of market stock prices is the one which exceeds the volatility of theoretical stock prices, this fact is not congruent with theory but as well it reflects real movements in stock market. Robert Shiller supposed in basis of these results that there is excess volatility in market stock prices relatively to prices given by theoretical asset pricing models. He investigated more about stock prices excess volatility, he firstly made a clear definition to excess volatility in stock prices and he assigned it to the over-ups and over-downs of stock prices than they are expected by theoretical stock prices and asset pricing models, then he used empirical tests to prove the existence of excess volatility anomaly in stock market. Moreover he evoked that excess volatility might be due to permanent activities of investors, they adjust permanently their forecasts in response to new information on possible anticipated events which may not occur in reality, and so the uncertainty surrounding the investors'

expectations and their dynamic reactions are the first responsible for stock market price excess volatility. Since the first discussions of Excess volatility, a lot of researchers have been interested in analyzing Excess volatility, sources and reasons behind it and factors that are responsible for high volatility of market stock prices relatively to theoretical prices. From 1981 till now, three big aspects in excess volatility have been the subjects of all research papers, one of them if it is not all.

The first aspect and the obvious one, is the excess volatility tests. The first concern of researchers after the appearance of this new concept was to show its existence and prove that all market stock prices are infected by excess volatility. Shiller 1981 and LeRoy and Porter 1981 have introduced the first papers that dealt with excess volatility bounds tests. They used the first generation of volatility bounds tests in which they were supposed to use either all stocks in the market or only market index. They use historical data of all US stocks whose financial data is available and cross section prices to test for the inequalities of stock prices volatility bounds. In their papers, they used stationarity hypothesis of dividends and they supposed uncorrelation between market stock prices series and expectations errors series. Normally, excess volatility bounds tests consist on the null hypothesis that the volatility of fundamental prices P_t^* is higher than the volatility of market prices P_t , and the alternative hypothesis is the violation of this inequality which reflects the excess volatility argument. Both Shiller and LeRoy-Porter confirmed the existence of excess volatility in their papers. Because of dividends stationarity hypothesis and the uncorrelation between market stock prices and disturbance term assumption, several researchers criticized their findings and made some remarks about the consistency of excess volatility tests used by Robert Shiller and LeRoy-Porter. Flavin 1983, Marsh, T. A., and R. C. Merton 1983 and Kleidon 1986 criticized the first generation excess volatility bounds tests and reported in their research papers some statistical properties of samples which may give biased results of excess volatility tests. The previous volatility bounds were established in basis of two conditions. The first condition is the uncorrelation between market price and forecast error, it was a necessary criterion to establish variance inequality but it was crucial to test this condition in data before doing variance bounds tests. The second condition was the stationarity of dividends series, before testing volatility bounds, it was necessary to test the stationarity of dividends series to make sure that dividends do not deviate largely from its trend. These two conditions are not always satisfied by the data used in empirical tests, especially when the samples are too small to generalize relevant empirical results; hence the previous results of variance bounds tests and empirical findings of excess volatility were considered biased. Moreover, Flavin 1983, Marsh, T. A., and R. C. Merton 1983 and Kleidon 1986 set that the small size of samples used by Robert Shiller 1981 and LeRoy and Porter 1981 biases their results because small sample variance downwards population variance and the smoother series are the higher downward bias infect prices variance. P_t^* is weighted sum of

dividends and since dividends are smooth series, this problem put greater downward effect in variance of P_t^* relatively to variance of P_t . It seems that these properties foster the violation of volatility bounds and the existence of stock prices excess volatility. To tackle the biases of volatility bounds tests Mankiw and al 1985, Campbell and Shiller 1987, and West 1988 established new inequalities to correct the previous biases and test volatility bounds without supposing the previous hypothesis of dividends stationarity and uncorrelation between error term disturbance and market stock prices that were necessary but no longer. Even though, after adjusting for the biases, researchers found the same results; the inequality exposes that the volatility of theoretical prices exceeds the volatility of market stock prices was also violated in the second volatility bounds tests generation. They also make the confirmation that market stock prices exhibit excess volatility and their variance exceeds the variance of theoretical prices.

After making sure the existence of Excess volatility in stock prices and showing that market prices are infected under all conditions by excess volatility, researchers moved to tackle the second aspect of Excess volatility, they became interested to explain the sources of excess volatility and explore factors behind the excess dispersion of market stock prices relatively to their theoretical prices deduced from theoretical asset pricing models. Researchers suggested different explanations to stock prices excess volatility. In the literature we find several research papers talking mainly about the excess volatility of stock prices, how market stock prices become more disperse than they are expected and how factors are contributing on injecting more volatility in market stock prices. We learn about many factors and components in economic finance and behavioral finance that are linked closely or less close to the excess volatility, to make the aspect of factors that contribute in stock prices excess volatility clearer I suggest to summarize all these factors in four essential ones than I will later explore each one of them in more details. In 1994, BENJAMIN EDEN and BOYAN JOVANOVIC suggested in their research paper that the market's assessment of the likelihood that some events will occur fluctuates in response to factors that are not included in theoretical asset pricing models; these factors are mainly responsible for the difference in volatility amplitude between market stock prices and prices from asset pricing models. We start with the first factor; it has been discussed in 1992 by James Dow. He reported that stock prices are always infected by uncertainty where agents in the market don't have enough information to expect future distributions of random variables, in our case they are distributions of dividends. This uncertainty is immeasurable and it is called Knightian uncertainty. James Dow shows when investors are not enough informed, their subjective distributions are very volatile and especially in case of Knightian uncertainty, thus stock prices would exhibit high dispersion relatively to their fundamental values dispersion. The second factor involved in stock prices excess volatility is learning factor. Allan Timmerman has

attempted in his study in 1996 to show the contribution of learning in creating stock prices excess volatility. He used a rational expectations model to construct a model for stock prices; he supposed that in each time investors use Recursive Least Squares (RLS) method as a way to simulate learning effect. As result, he got a stock price model where stock prices are sum of dividends term and another term under learning effect, he split the variance of stock prices into dividends variance and variation term generated by learning effect and he tested empirically these results to show the effect of learning in amplifying stock prices volatility relatively to dividends volatility. In 1991 Cochrane John evoked in his research paper a third factor responsible for stock prices excess volatility; it is investors' behavior irrationality. He suggested that irrational pessimism and irrational optimism, noise trading, feedback trading, speculative enthusiasm or also frequent changes in investors' psychology may all be behind stock prices excess volatility. In 1996 Paul H Kupiec discussed in his study the impact of short term speculative trading volume and he assumed that it may be the source of excess volatility, since in the market speculators don't aim to own, buy and hold assets, their objective is to realize profit, buy when it is down, sell when it is up and win the capital gains. This high frequency and excess trading provoked by speculators generate excess volatility. Moreover, PHILIPP KARL ILLEDITSCH talked in 2011 tackled portfolio inertia phenomenon, exogenous reasons pushing investors to act irrationally and aggressively as feedback to true or fake news. In 1997, George Bulkley and Richard Harris used a model in their study to test for the link between excess volatility and investors' behavior irrationality and show the contribution of this latter in amplifying stock prices dispersion. Among the four factors that contribute in stock prices excess volatility, the fourth one is reaction to information in rational behavior across overreaction and overconfidence. In 1985, De Bondt and Thaler suggested that excess volatility may be result of investors' overreaction to new information whereas in 2003 Jose´ A. Scheinkman and Wei Xiong dealt in their research paper with the effect of overconfidence in creating excess dispersion of stock prices. They used a model to prove that when investors have overconfidence on public information, they believe that their expectations based on the available information are more accurate than other investors' expectations. Overreaction to new information and overconfidence are considered as rational behaviors, Carsten K. Nielsen in 2007 and Carl R. Chen, Peter P. Lung, F. Albert Wang, in 2013 justified in their research studies that overreaction and overconfidence are rational behaviors and they are the main generator of heterogeneous beliefs and un-common expectations even if information is common. As it is obviously seen, these factors separately or together lead to excess dispersion in stock prices relatively to their fundamental values.

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CHAPTER 2 – EXCESS VOLATILITY BOUNDS TESTS

Before talking about excess volatility as common risk factor, researchers have been pre-occupied by the evidence of its existence in stock market. Excess volatility bounds tests are empirical tests used to test for the inequalities of market and theoretical stock prices volatility and show the evident existence of excess volatility in stock market using cross section historical data.

I. THE FIRST GENERATION OF EXCESS VOLATILITY BOUNDS TESTS

In basis of efficient markets model: asset price is defined as the sum of the present value of future dividends: $P_t^* = \sum_{k=0}^{\infty} \frac{d_{t+k}}{(1+R)^{1+k}}$ and as future dividends are not known at time t except the next one; P_t^* will then be characterized as random variable with mean $E_t(P_t^*)$ and $Var(P_t^*)$. We define the expected price in the market as follows: $P_t = E_t(P_t^*)$ It is the mathematical conditional expectation on available information at time t. We may notice obviously that the volatility of P_t^* must be greater than the volatility of P_t since the second is the expected value of the first, this inequality is what we call volatility bounds as the volatility of theoretical stock price is the great bound of the volatility of market stock price, but this inequality has been violated over empirical tests and this violation has been expressed as excess volatility when the volatility of market stock prices exceeds the volatility of theoretical stock prices.

In his research paper in 1981, Shiller explained the evidence from volatility bounds using dividends asset pricing model as it is shown above. The stock price P_t^* and its expectation $P_t = E_t(P_t^*)$ are linked as the following formula shows: $P_t^* = P_t + U_t$ where U_t is the forecast error. Shiller supposed in this study that the forecast error U_t and the expected stock price P_t are uncorrelated and hence $COV(P_t, U_t) = 0$, he also supposed dividends series to be stationary. These hypotheses were necessary to consider that $VAR(P_t^*) = VAR(P_t) + VAR(U_t)$ and establish the volatility bound $VAR(P_t) < VAR(P_t^*)$ which means that by definition the volatility of dividends stock prices must be greater than the volatility of market stock prices. Volatility bounds tests are dedicated to test the inequality we mentioned so far; we consider the null hypothesis is that the inequality is satisfied and the alternative hypothesis is the opposite of the inequality which is equivalent to accept that market stock prices exhibit excess volatility relatively to theoretical stock prices.

Researchers conducted in their research papers the first generation of volatility bounds tests as we summarize below the steps to proceed for these tests in a clear algorithm: The first step in volatility bounds tests is to gather the data, historical dividends' series and historical market stock prices' series between the starting and ending dates that we choose according to the available data.

As dividends asset prices P_t^* extend to infinity, stock prices based on the present value of dividends are never observed without error. But if the sample is large enough, the approximation to P_t^* is less erroneous. From the sample, we choose the terminal value of fundamental price P_t^* as the final value in dividends series, since it is not observable, we consider the average of detrended market prices over the sample as a proxy for the final value of theoretical stock prices, then we compute $P_t^* = \bar{\gamma} * (P_{t+1}^* + d_t)$ for each year by working backward from the last value in the final year to the starting year using dividends series, $\bar{\gamma} = \frac{(1+g)}{(1+r)}$. We use detrended price in the terminal value to remove growth term because the terminal value in dividends stock prices is considered as the final value where growth or trend of growth must not be considered, detrended prices are set to $P_t^{det} = (1+g)^{-t} * P_t$. After computing P_t^*, we move to compute variance of P_t^* and variance of market prices series. For each stock in the market we compute both variances then we compute the averages over the stocks sample. We are interested in all stocks that exist in the market because excess volatility infects all stocks; hence we do cross section study of volatility bounds tests. After getting values of the average over stocks of the volatility of market stock prices and the average over stocks of the volatility of dividends stock price we compare both variances and do empirical tests considering the null and the alternative hypothesis that we mentioned above.

Shiller and LeRoy-Porter found in their research paper that the null hypothesis considering the volatility bounds is rejected and the variance of market stock prices is five times greater than the variance of ex-post prices using dividends. This finding supports the excess volatility hypothesis and confirms that market stock prices exhibit high dispersion relatively to dividends stock prices.

II. CRITICISM AND ANOMALIES OF THE FIRST GENERATION OF EXCESS VOLATILITY TESTS

The first generation volatility bounds tests have been criticized by different researchers in their studies namely Flavin in 1983, Mankiw and all 1985 and Kleidon in 1986. They reported some anomalies of these tests. From one hand, before doing these tests it was necessary to make sure of the first hypothesis and test that the uncorrelation between market prices and forecast errors assumption is satisfied by the data. The hypothesis of the uncorrelation between dividends series and disturbance term series was necessary to establish the first generation of volatility bounds tests but this assumption is not always valid. The second hypothesis was stationarity of dividends series which is also not always satisfied by the data. From the other hand, the sample variance of prices' series downwards the population variance and the smoother are series the greater downward effect is. Since P_t^* is weighted sum of dividends and these latter are smooth series, P_t^* is smoother than P_t,

thus downward effect in the variance of P_t^* is greater than downward effect in the variance of P_t which amplifies and fosters the excess volatility evidence.

III. THE SECOND GENERATION OF EXCESS VOLATILITY TESTS

After a set of criticism, Mankiw and all proposed in 1985 the second generation of volatility bounds tests which are unbiased in small samples and they do not require assumption of dividends stationarity neither assumption of uncorrelation between market stock prices and forecast errors.

In the new generation volatility bounds tests, researchers used the same theoretical asset pricing model which gives stock prices as the present value of future dividends, they removed the hypothesis of dividends series stationarity because it is no longer necessary to establish the volatility bounds, moreover they deleted also the hypothesis of the uncorrelation between market stock prices and forecast errors since they do not need this assumption to get the volatility bounds inequality. As, dividends stationarity hypothesis was eliminated, the inequivalence of the downward effect of population variance over sample variance between market stock prices and theoretical stock prices was then eliminated. Hence the empirical tests of the volatility bounds do not face the problem of small sample size and computations are not biased. Instead, researchers used other properties to establish the volatility inequalities. They introduced a new concept to establish the volatility inequalities; they consider naïve forecasts F_t as worse forecasts relatively to the standard expectations E_t. The naïve price P_t^0 is computed by the same way as P_t but this time by considering naïve forecasts in the present value of future dividends instead of rational forecasts, we got then $P_t^0 = F_t\ (P_t^*)\ =\ \sum_{k=0}^{\infty} \frac{F_t(d_{t+k})}{(1+R)^{1+k}}$.

Market stock price P_t and theoretical stock price P_t^* remain as in the previous analysis.

Based on the consideration of naïve price and naïve forecasts, Mankiw and all established new inequalities; since $E(P_t^* - P_t^0)^2 = E(P_t^* - P_t)^2 + E(P_t - P_t^0)^2$ they obviously deduced that the expectation of squared difference between the expected price and the naïve price is lower than the expectation of squared difference between theoretical price and naïve price:

$E(P_t - P_t^0)^2 < E(P_t^* - P_t^0)^2$. As we are aiming to test the existence of excess volatility in stock market, we must consider tests in the worst conditions-naïve forecasts- fostering the opposite of the excess volatility; to make sure that excess volatility does not exist because we missed some specifications or we considered some conditions that are in favor of the existence of excess volatility as it has been criticized in the first generation of volatility bounds tests. The idea behind the new inequality is to compare variance of time series around naïve forecasts instead of their means. Using this property we assume simultaneously that the first assumption of the uncorrelation between forecast error and market price is not necessary, stationarity dividends assumption is useless,

downward effect problem does not infect tests results and the volatility bounds tests are made due to this inequality which is equivalent to the first one.

The empirical tests are done considering equivalent hypothesis to the previous ones; the null hypothesis is that the data satisfied the new inequality $E(P_t - P_t^0)^2 < E(P_t^* - P_t^0)^2$ and the alternative hypothesis is that the inequality is violated and by that we accept that market stock prices exhibit high dispersion relatively to theoretical stock prices. To do these tests, we firstly collect data for market prices' series P_t as previously in the first generation tests from the starting year to the ending year, the reference period is choosing according to the available data in the market, then we choose naïve forecasts $P_t^0 = \sum_{k=0}^{\infty} \frac{F_t(d_{t+k})}{(1+R)^{1+k}}$ where the forecasts are completely myopic $F_t(d_{t+k}) = d_{t-1}$, by this consideration we give high magnitude to the difference between theoretical price and naïve price relatively to the difference between the expected price and naive price, and so we foster the high volatility of theoretical prices relatively to the volatility of market stock prices. To avoid the second assumption about stationarity of dividends series, we compute P_t^* during empirical tests using the following formula $P_t^* = \sum_{k=0}^{T-T-1} \frac{d_{t+k}}{(1+R)^{1+k}} + \frac{P_T}{(1+R)^{T-t}}$ where T is the final date, P_T is the final price supposed to be in the final date. As in the previous empirical tests for volatility bounds, we refer to the terminal value P_T the average of detrended market prices.

After representing the three series: P_t^0, P_t^* and P_t we compute both terms of the inequality for all stocks in the market, we compute the averages across stocks and we compare $E(P_t - P_t^0)^2 to\ E(P_t^* - P_t^0)^2$.

Even after adjusting for the anomalies, the hypothesis of the inequality has been rejected and so the excess volatility of market stock prices relatively to dividends stock prices is satisfied.

For both the first generation and the second generation volatility bounds tests, we can use market index to check for the inequalities but we can also use all the assets listed in an exchange market by computing market stock price and dividends stock price volatilities over the whole sample for each stock then compute the averages across all the available assets to check the inequalities only for the averages.

CHAPTER 3 – RESPONSIBLE FACTORS FOR STOCK PRICES EXCESS VOLATILITY

The previous chapters have shown that excess volatility exists in stock market from many decades, and it remains an important issue to tackle since it has huge impact on the expectations and behaviors of investors in the market, on price movements and market equilibrium. It was very important in the first studies dealing with excess volatility to prove its evidence and existence in stock market, but it is as much important to explore the sources of excess volatility and to understand where it comes from. Before talking about its impact and how it affects the investment decisions of investors, we must discuss the factors that are responsible for excess volatility and find explanations to the high dispersion of market stock prices. BENJAMIN EDEN and BOYAN JOVANOVIC suggested in their research paper in 1994 that the market's assessment of the likelihood that some events will occur fluctuates in response to factors that are not included in asset pricing models. From this remark, we notice that daily impacts of set of factors are not considered in theoretical stock prices while their actions in market stock prices movements are obviously noticeable. If we think out to the effect, we notice that the absence of these factors would create inequivalence between theoretical prices and market stock prices and more specifically they would enlarge the difference in the magnitude of fluctuations between both prices. From these first remarks, we expect that there are factors behind the excess dispersion of market stock prices and there are explanations must be clarified to admit the evidence of stock prices excess volatility. Evident observations are not sufficient to explain the excess volatility; we must be more accurate and consistent in considering it, especially if we aim to use it as a common risk factor in equity risk premium models. For this reason, in the remainder of this chapter we will discuss in more details the impact of four main factors that are responsible in stock prices excess dispersion: Knightian uncertainty and risk aversion, Learning, investors' behavior irrationality and then rational reaction to information through overconfidence and overreaction. I suggest some concrete examples from stock market to explore the link between these factors and Excess Volatility, I move to expose some models which were proposed in the previous studies to prove these links then I end up by comments on these models regarding the present study.

I. KNIGHTIAN UNCERTAINTY

A. Examples

The first factor was been suggested in 1992 by James Dow, he reported that the profitability of companies depends on several long term factors such as political factors, economic factors and governmental factors which are extremely difficult to predict, hence stock markets are always

infected by uncertainty that is called Knightian uncertainty. I set below some concrete examples showing the Knightian uncertainty and its effects on stock prices volatility:

- Economic states: The more economic variables are volatile the less Equity risk premium and prices are stable. In economic expansions or recessions and economic crisis, economic variables are more volatile than it is expected and then prices volatility exceeds the expected volatility.
- Political reasons: some political events like Brexit in 2016 and American elections at the end of 2016 have injected a lot of uncertainty in financial markets and especially in stock market. The stocks and companies that were the most infected by this uncertainty were the international companies and firms that their services (as airline companies) or their products are based on abroad exchanges between countries. This uncertainty surrounding the invoked firms has put more volatility in their stocks' prices.

Governmental reasons: Expected changes in monetary policy, budgetary policy, fiscal policy and government policy increase stocks volatility as investors do not have information about what would be the future decisions of the government and how the market would react in response to these changes.

B. Model

Knightian uncertainty is immeasurable risk where agents in the market don't have enough information to expect future distributions of random variables, which are in our case distributions of dividends. James Dow shows in his research paper that in presence of Knightian uncertainty investors react rationally, they use the available information to construct their subjective distributions of probability. Because they are not enough informed, subjective distributions of investors are very volatile and especially in case of total absence of information and Knightian uncertainty, thus stock prices would exhibit high dispersion relatively to their fundamental values.

Knightian uncertainty is one of other factors that create excess volatility, this finding is plausible from what it is so far said, but James Dow used in his research paper a model to show the contribution of Knightian uncertainty in causing stock prices excess volatility. We can use the same model to show how Knightian uncertainty creates excess volatility. The model is an experimental model, where assets are traded in only three periods:

Table 1 : Trading sessions in the experiment

Period	Period 1	Period 2	Period 3
Expected stock price P	P0 = E(V)	P2 = E (V | Information)	P3 = V

V is the value of stocks from dividends asset pricing model

Information is the available information

The price in the last period must be confounded to the value of dividends

The price in the first period is expected without any information because investors don't know any information about process dividends in the first period

The price in the second period is expected conditionally to the available information because investors gather some information about dividends process from trading sessions.

Agents have information about how much dividends worth in each state, but they don't know dividends distributions

Table 2 : Dividends Process

State	State 1	State 2	State 3
Dividends	D1	D2	D3

To model the actual distribution and the subjective distribution of agents for dividends process, he considers π the actual probability and μ the subjective probability. After the experiments, James Dow used the results of multiple iterations to compute variance of market price and variance of dividends price, he computed market price from the results of the experiment and dividends price from dividends process imposed in the experiment.

C. Is this model consistent with our study

The market price in James Dow model is obtained under the assumption of Knightian uncertainty in the experiment hence it reflects the impact of Knightian uncertainty. Variances are computed using the actual distribution to remove any other impact and isolate only the impact of Knightian uncertainty. The dividends process in known but its distribution is not, even though investors in the experiment do not use standard learning process to expect distributions they use their own subjective probabilities which means that the effect of learning is removed from this model. Moreover the computations of prices and variances are based on averages prices over investors' expectations; this consideration is relevant with our interest to remove the effect of overconfidence between investors. As well as the behavior of investors to rely on their subjective probabilities when no information is available about distributions of the states is a rational behavior and this is also consistent with the isolation of Knightian uncertainty factor from investors irrationality factor. The results that James Dow found using all these specifications supports the excess volatility since the

market price volatility exceeds the dividends price volatility and confirms the contribution of Knightian uncertainty in stock prices excess volatility without considering the effect of the other factors.

II. LEARNING

A. Examples

The second factor involved in stock prices excess volatility is learning factor. Allan Timmerman has attempted in his study in 1996 to show the contribution of learning in creating stock prices excess volatility. He explored this model under rational expectations. But before exposing the model I suggest the below examples to illustrate learning factor:

- Exchanging information between informed and uninformed investors throughout trading sessions of the same stocks serves them to learn more about these stocks in the market to expect their future prices, the prices set by investors are volatile in response to new information
- Firms' disclosures: earnings announcement, accounting data disclosure and annual reports are all tools that analysts use to learn more about stocks and extract more accurate information about the future prices, the disequilibrium of information and learning flows sets excess volatility in stock prices.

B. Model

Allan Timmerman specified in his model that rational investors expect stock prices by expecting the values of the unknown parameters in the model. As a rational behavior, investors use the available information to estimate these parameters. Allan Timmerman supposed that investors must include the information they learnt in their future expectations and analyses, hence in each time investors use Recursive Least Squares (RLS) method as a way to integrate the new information they got from learning. He established a stock price model where stock prices are sum of dividends term and another term representing the effect of learning on stock price deviations. Finally, he split the variance of stock prices into dividends variance and variation term generated by learning effect and he tested empirically these results to show the effect of learning in amplifying stock price volatility relatively to dividends volatility.

The model starts by setting ordinary dividends asset pricing model: $P_t = \frac{1}{r+1} E(P_{t+1} + d_{t+1} |\Omega_t)$ where Ω_t is the available information at time t and r is one period required return which is assumed to be constant. Then the current dividends process is supposed to be known, all agents know the dividends process but they don't know the true value of model parameters.

$d_{t+1} = \mu + \gamma(t+1) + \rho\, d_t + \varepsilon_{t+1}$ is dividends process where μ is a constant, γ is dividends drift, $\mu + \gamma(t+1)$ is dividends trend and ρ is dividends persistence. With all these specifications the stock price would be computed as following $P_t = \frac{\rho\, d_t}{1+r-\rho} + \frac{1+r}{r\,(1+r-\rho)}(\mu + \frac{1+r}{r}\gamma + \gamma\, t)$, the vector of parameters $\beta^d = (\mu + \gamma,\ \gamma, \rho)$ gives the true values of $\mu,\ \gamma\ and\ \rho$, dividends for time t are defined by $D_t = X_t * \beta_t^d + \xi_t$. Agents don't know the true values of β^d they only expect them. To explore learning effect, in this model agents use Recursive Least Squares (RLS) method to expect values of parameters. This method consists on finding estimate point for β^d , they estimate parameters for each time t using all information that they learnt from the previous iterations. At date t they use past data of dividends series $D_t = (d_t\,, d_{t-1}\,, d_{t-2}\,, \dots\dots, d_2\,, d_1)$, error series $\xi_t = (\varepsilon_t\,, \varepsilon_{t-1}\,, \varepsilon_{t-2}\,,, \varepsilon_2\,, \varepsilon_1)$ and the matrix $X_t = (x_t\,, x_{t-1}\,, x_{t-2}\,, \dots\dots, x_2\,, x_1)$ Where $x_t = (1, t,, d_{t-1})$. After finding estimation for $\widehat{\beta_t^d}$, they compute stock price at time t $\widehat{P_t} = \frac{\hat{\rho}\, d_t}{1+r-\hat{\rho}} + \frac{1+r}{r\,(1+r-\hat{\rho})}\left(\hat{\mu} + \frac{1+r}{r}\hat{\gamma} + \hat{\gamma}\, t\right)$. After getting $\widehat{P_t}$ which represents otherwise the expected price or the market stock price P_t , James Dow had to find values of theoretical stock price P_t^* , he regressed the whole dividends series sample using the following equation $d_{t+1} = \mu + \gamma(t+1) + \rho\, d_t + \varepsilon_{t+1}$ to find the sample parameters, he considered whole sample values as the true values of model parameters and we use them to compute the real values of stock prices P_t^* under rational expectations besides $\widehat{P_t} = P_t$ series under learning effect. From the expression of P_t, he split stock price into two terms, the first one represents rational expectations using dividends and true value of dividends persistency ρ and the second one depends on estimate points of model parameters under learning effect. This expression permits to divide the variance of stock prices into variance due to dividends volatility and variance due to learning effect. From this fact it is trivial that the variance of market stock prices under learning exceeds the variance of theoretical stock prices because of the supplementary term of learning effect. Moreover, the empirical results for large sample show that the stock prices under learning effect $\widehat{P_t}$ show greater volatility relatively to variance of stock prices P_t^* only under rational expectations. This finding supports the contribution of learning effect in creating stock price excess volatility.

C. Is this model consistent with our study

This model does not compare the variance of market stock prices to the variance of theoretical prices; it simulates market stock prices under only learning effect. In our study we aim to isolate the effect of each factor separately to make sure that they all contribute in stock prices excess volatility without biases. In this paragraph we expose only the effect of learning. The effect of Knightian uncertainty factor is removed from this model, since the value market prices are not picked from the data in the market and they are computed only in basis of dividends process and the estimates of

model parameters, thus there is no uncertainty about political or economic conditions neither about the states and their distributions. The same expectations of prices $\widehat{P_t}$ are used to compute the values supposed to be assigned to the market stock prices; this means that the overconfidence effect between investors does not take place in this model. And at last, the behavior of investors in the execution process of this model is rational, as they use RLS model to expect the future stock prices when no other information source exists. Hence neither, irrationality of investors behavior factor, Knightian uncertainty factor nor overconfidence effect are present in this model and this latter test only for the effect of learning in contributing in excess dispersion of stock prices.

III. INVESTORS' BEHAVIOR IRRATIONALITY

A. Examples

Many discussions are taken on behavioral finance and behavioral economic, these both fields have been enhanced in a rapid rate in the market while they were not the basis of finance. This increasing interest on investors' behavior studies shed the light on the importance of behavioral finance in explaining different anomalies and facts which remain unexplained. In this basis I chose to look over investors' behavior and emphasis how the investors' reactions and considerations take part in explaining high dispersion of market stock prices relatively to dividends stock prices. Hence, the third factor I suggest as one of the factors responsible for stock prices excess volatility is investors' behavior irrationality. Investors have rational and irrational behavior, in fact not only their irrational behavior generates large fluctuations which remain unexplained by movements in dividends but also their rational decisions do. In this part we would focus only on irrational behavior and how it contributes in creating excess volatility in stock prices. I set below some forms and examples of excess volatility to show how they foster the over-ups and over-downs of prices in the market:

- Before we start the examples, I want only to mention that not only irrational investors, that are in our minds investors who are uninformed and they take investment decisions without getting reference to the relevant information, but also rational investors have irrational behavior which is not always based on their backgrounds and their rationality in interpreting information.
- Irrational pessimism and optimism and frequent changes in investors' psychology are among exogenous reasons generating excess volatility; investors may panic through over pessimism and sell stocks without any relevant justification (prices dramatically collapse). For example if investors remark non-ordinary collapse in stock prices, they believe that this is because the stock is not going good and the market is negatively reacting to this stock, they panic and they start selling in huge volumes their stocks and then prices collapse more and more away

from their fundamental values. Or they may buy stocks in huge volumes only because they feel optimistic towards some stocks; prices dramatically rise and go up away from their real values, between dramatic ups and downs of stock prices around their fundamental values, stock prices become more volatile than they should be.

- Feedback trading: aggressive reactions of investors to some unusual events, true or fake news creates excess fluctuations in stock prices and thus excess dispersion. Sudden news amplifies the effect of investors' reaction to information and makes changes in prices more dramatic. For instance, for stocks with positive betas: if investors receive disappointing news about the market from trading, they aggressively sell their stocks since their assets are positively correlated to the market and they think they will undergo the same collapse movements as the market. Same for stocks with negative betas, if investors receive good news about the market from trading, they aggressively sell their stocks, as their assets negatively correlated to the market and they think that the market is going good so they prefer to sell their stocks even in low prices before their portfolios go worse, thus prices exhibit dramatic change.
- Noise trading: investors may trade stocks for other reasons namely liquidity and hedging objectives, this creates noise in the market and generates ambiguous fluctuations in stock prices independently whether stock performances and expected dividends are bad or good, these irrational fluctuations make prices more disperse than expected prices over discounted dividends model
- Portfolio inertia and ambiguity: Investors that incur portfolio inertia they make quick reactions to hedge their portfolios without paying attention to other data namely stock performances, accounting and financial data. They may react aggressively to adjust their portfolios and make them hedged back. This reaction sends fake and ambiguous news to the market and impact prices' fluctuations to be more frequent and larger than movements of fundamental values.
- Speculative trading and ambiguity: in speculative trading, investors don't aim to buy and hold assets, their objective is to realize profit, buy when it is down, sell when it is up and have the capital gains. Since they are aiming to collect capital gains, they can trade for several times a week the same stock, buy it when it collapses and sell it when it goes up. If they are very skilled, they even may trade stocks several times the same day to take profit from intraday volatility and daily mean reversion property of stock prices. These actions of speculators are not rational, they don't really focus on information because they don't aim to hold stocks, and their only objective is to seize the ups and downs opportunities of stock prices.

Moreover they may also find strategies to foster these ups and downs in their favor. The high frequency and excess trading provoked by irrational behavior of speculators generate excess fluctuations in the market and so excess dispersion of market stock prices as well as they inject false information and ambiguity in the market. Investors who are not informed use this information to set prices more or less deviated from their fundamental values but surely more volatile than they should.

B. Model

These explanations and examples are not sufficient to prove definitely the contribution of irrational behavior in setting excess volatility in stock prices. As we mentioned above in the first chapter of review part, George Bulkley and Richard Harris in their research paper in 1997 used a model to show the contribution of irrational behavior of investors in adding supplementary dispersion in stock prices. A cross section model is used to test for the contribution of the third excess volatility factor. This model suggested by George Bulkley and Richard Harris consists on the following steps:

- We gather data for a large number of companies listed in the same exchange market, we collect accounting data (realized annual earnings of these companies), financial data (annual firm size, monthly returns) and analysts' forecasts (are available in IBES databases) of these companies' earnings. We need to make sure that all this information is publically available for all the companies in the sample and for each year in the horizon period we choose to do the study on.
- We compute realized earnings growth over five years for all companies in several years, we compute analysts' forecasts of earnings growth over five years of the same companies in the same period
- We compute the correlation between realized earnings growth and analysts' forecasts of earning growth, we test for the nullity of correlation coefficient where the null hypothesis is that the coefficient is null and the alternative hypothesis is that the coefficient is different from zero. In George Bulkley and Richard Harris research paper, the null hypothesis could not be rejected which means that correlation coefficient is not significantly different from zero and hence confirms the linear uncorrelation between realized earnings growth and analysts forecasts and in other words confirms the irrationality of investors behaviors in expecting companies earnings growth .
- We then do another test for the correlation coefficient of analysts' forecasts of earnings growth over five years and realized earnings growth over five years, but in this time the null

hypothesis is that the coefficient is equal to one and the alternative hypothesis is that the coefficient is less than one. In George Bulkley and Richard Harris research paper, the null hypothesis was rejected which means that the correlation coefficient is significantly less than one and in other words when forecasts are very high, real earnings are lesser and so expectations overestimate earnings. When expectations are very low, real earnings are greater and so expectations underestimate earnings. These overestimations and underestimations give underpriced and overpriced assets, these prices are deviated from their fundamental values and so over-disperse; it is stock prices excess volatility.

- We look for systematic determinants which may affect analysts' expectations and impact their decisions, but we need to make sure that information about these determinants is publically available at the date of analysts' forecasts.
- After finding the right determinants, which are firm size, lagged earnings and betas. We regress in the first step realized earnings growth on these three factors and we regress in the second step analysts' forecasts of earnings growth on the same determinants to compare both regression coefficients and deduce some results. In George Bulkley and Richard Harris research paper, the regression coefficients of the two regressions are completely different from each other, the regression coefficients in analysts' forecasts are very deviated from real values in realized earnings growth regression, this means that analysts misinterpret the impact of these variables (firm size, beta and lagged earnings) on future earnings and so this prove an irrationality in expecting earnings or earnings growth.

Using the above model of George Bulkley and Richard Harris, researchers prove irrationality of analysts in expecting earning and earnings' growth. After that, they construct two portfolios, the first one is equally built using 5% of stocks which have the lowest forecasted EG and the second portfolio is equally built using 5% of stocks which have the highest forecasted EG. They found high realized returns for the first portfolio and low realized returns for the second portfolio.

C. Is this model consistent with our study

George Bulkley and Richard Harris used the above model to prove the presence of irrationality in investors' expectations in the market then they constructed two investment portfolios to show the link between analysts' forecasts of earnings and actual prices or returns. The portfolio with the lowest expected earnings realizes the highest returns and the portfolio with the highest expected earnings realizes the lowest returns. This is consistent with the fact that there is a link between analysts' expectations and prices or returns in the market. Irrational earnings expectations appear to play a substantial role in explaining the excess dispersion of stock prices. When analysts' forecasts of

earnings are relatively higher than the true values, prices are relatively higher and returns relatively lower. When analysts' forecasts of earnings are relatively lower than the fundamental values, prices are relatively lower and returns relatively higher. Prices are sometimes overvalued and other times undervalued; this mispricing is due to analysts' forecasts irrationality. For instance, when a firm announces to increase its leverage, investors think that the firm is facing some financial problems, they may suppose that it is in its way to go bankrupt, they may also think that its projects are not going well and they are not generating enough profit to pay for its engagements while the firm has announced to increase its debt because it would undertake more successful projects. But these irrational expectations of investors push them to make poor earnings' forecasts, they express that they are not ready to pay high prices for such risky assets, hence prices in the market collapse but latterly prices go up which invokes excessively high returns in the market. From these explanations we conclude that prices can be excessively high or low relatively to their fundamental values because of irrational behavior. It is the contribution of investors' irrationality in creating excess dispersion in stock prices.

The principle in our study is to isolate the effect of each factor and test its contribution in excess volatility independently to the other factors. For this model, the objective is testing the effect of investors' behavior irrationality, it started measuring correlation between forecasts and realized earnings, the result was zero which means that there is irrationality in investors' expectations, but in this case we don't care about investors' overconfidence factor because from one hand overconfidence is seen as rational behavior and from other hand forecasts of earnings are taken as average of forecasts over all analysts without caring about the difference in their expectations. The regression of analysts' forecasts on firm size, beta and lagged earnings growth is done as cross section over stocks and over time, as we consider that there is no time for investors to consider results of the previous regressions. They could neither learn from results deduced from realized earnings regression nor from forecasted earnings regression, they lose the opportunity to adjust their methods and expect the future earnings by focusing on the regressions' independent factors proportionally to their regression coefficients. This specification means that we don't take learning into consideration and its effect is removed in this model. Moreover, in this study, we consider only earnings of companies and expectations of analysts of these earnings. In general the results of this study are linked to what analysts forecast about companies earnings and not what they forecast about economic, government and political states. Thus we can say that Knightian uncertainty has no effect here. After making sure that the effects of all other factors are neglected in this model, we accept that this model is sufficient in our study to test for the impact of investors' behavior irrationality independently to the other factors.

IV. OVERCONFIDENCE

A. Examples

After discussing the impact of Knightian uncertainty and risk aversion, learning and investors' behavior irrationality in generating excess dispersion in stock prices, the fourth and the last factor responsible for excess volatility is overconfidence and overreaction to new information under rational investors' behavior.

- Several researchers supported that overconfidence is totally a rational behavior. For example, when investors receive new information, they use their own analysis skills and backgrounds to deduce interpretations from information and build future expectations; it is rational that investors rely on their analysis results and take decisions in basis of their own beliefs. The information disclosed and received by agents in stock market may be exactly the same and common between all investors, however, they may make diverse interpretations of information and get diverse expectations. This finding pushes us to conclude that when investors in stock market rely on their own beliefs to interpret information, they generate heterogeneous beliefs about dividends growth and these fluctuations of rational subjective investors' beliefs on signals create large fluctuations in stock prices regardless real movements in dividends. Thus overconfidence and heterogeneous beliefs are also responsible for excess volatility of stock prices.
- To illustrate how overconfidence contributes in stock prices volatility, we explore a frequent example. Suppose that investors have overconfidence in their own beliefs and interpretations of public information, they believe that their expectations are more accurate than other investors' expectations and so even if new information is exchanged between all investors through trading sessions, they make their investment choices in basis of their own beliefs. They may buy stocks because they see great volumes were bought and even if they see another signal contradicting their beliefs they keep their investment decisions which are based on insecure signals. Or they sell stocks because they see great volumes were sold without investigating the real reasons behind the traded stocks' volumes. They overestimate the precision of their knowledge, they think their expectations are the best ones and others are mistaken, hence they trade and set over or under reacted prices in the market. These overestimations and underestimations of stock prices create excess fluctuations and excess volatility of stock prices relatively to dividends volatility.
- This evidence we can observe it in bubbles and crashes phenomena which Robert Shiller talked about. When prices exceed their fundamental values, investors keep giving big values to the expected prices and prices keep moving up and up away from their fundamental

values, it is a bubble phenomenon but when they become conscious of the fundamental values of assets, prices collapse and keep moving down and down away from their fundamental values, it is a crash phenomenon. These bubbles and crashes are the best illustration of excess volatility of stock prices.

B. Model

The above examples illustrate how overconfidence in investors takes action in creating high dispersion in stock prices relatively to dividends stock prices, but we need concrete model to prove that. In the next part we explore a model to prove the contribution of overconfidence in stock prices excess volatility relatively to their fundamental values. This model has been developed by Jose´ A. Scheinkman and Wei Xiong in 2003. To start this model they precise some specifications:

- One risky asset: in this study they consider that all investors are risk neutral, hence they don't care about specific risk and noises. As result they don't ask for specific risk premium and so the procedure is the same for all the assets
- Short selling is not permitted to avoid the effect of speculative trading and isolate overconfidence effect on excess volatility
- r is the rate to lend and borrow money for all agents, it is the same discount rate for all agents
- All variables they use in this study are Gaussian
- Transaction fees and tax are taken into consideration as direct transaction costs, c represent these costs, it is what the seller pays for one unit of the sold asset

The model suggested by researchers consists on the following steps:

- Dividends process is defined by: $dD_t = f_t\ dt + \sigma_D\ dZ_t^D$ where f_t is the fundamental variable of dividends changes, it is not observable but it satisfies a mean reversion process $df_t = -\lambda\left(f_t - \bar{f}\right)dt + \sigma_f\ dZ_t^f$ where f_t is moving around its run long trend, when it exceeds its mean, it must go down and when it collapses under its mean it must go up at mean reversion speed λ. The second term is f_t changes variance multiplied by standard wiener process, it is noise term. σ_D is dividends' changes volatility and dZ_t^D is standard wiener process $dZ_t^D = \varepsilon\sqrt{dt}$ and ε follows standard Gaussian distribution. The term $\sigma_D\ dZ_t^D$ represents noise in dividends process.
- Agents' beliefs process: they consider two sets of investors group A and group B to exhibit overconfidence between both. Because of noise term in dividends process, investors cannot observe the fundamental value of dividends process f_t from dividends observations. Agents

A and agents B consider their own beliefs or signals on dividends process. Signal S^A of group A satisfies the process: $dS_t^A = f_t\, dt + \sigma_S\, dZ_t^A$ Signal $S^B$of group B satisfies the process: $dS_t^B = f_t\, dt\, \sigma_S\ \ dZ_t^B$. The wiener processes Z^A, Z^B, Z^f and Z^D are mutually independent

- Overconfidence: Both groups believe that their signals S^A and S^B are the right ones, they believe their signal innovations are correlated to the fundamental value and ϕ is the correlation coefficient which they consider the overconfidence parameter. Group A signal S^A satisfies $dS_t^A = f_t\, dt + \sigma_S\, \phi\, dZ_t^f + \sigma_S \sqrt{1-\phi^2}\, dZ_t^A$ and similarly group B signal S^B satisfies $dS_t^B = f_t\, dt + \sigma_S\, \phi\, dZ_t^f + \sigma_S \sqrt{1-\phi^2}\, dZ_t^B$. When the correlation ϕ between agents' beliefs and fundamental value of dividends process increases their overconfidence increases, that's why they consider ϕ as overconfidence parameter
- Agents' beliefs evolution: Agents' beliefs are all public information; each group has access to other group beliefs. Agents try to find a solution to the process of their beliefs S^A and S^B; the evident one is a stationary solution with initial conditions. Agents' beliefs are Gaussian variables with the variance $\gamma = f(\lambda, \phi, \sigma_D, \sigma_f, \sigma_S)$ for both groups and means $\widehat{f_A}$ for group A and $\widehat{f_B}$ for group B. The means $\widehat{f_A}$ and $\widehat{f_B}$ are also mean reverting since the process f of fundamental value does. They mean revert in response to three surprises. For group A, there are dividends surprises to group A W_D^A, group B surprises to group A W_B^A and group A own surprises. $\widehat{f_A}$ Mean of agents A satisfies $d\widehat{f_t^A} = -\lambda\left(\widehat{f_t^A} - \bar{f}\right)dt + \frac{\phi\, \sigma_f \sigma_S + \gamma}{\sigma_S}\, dW_A^A + \frac{\gamma}{\sigma_S}\, dW_B^A + \frac{\gamma}{\sigma_D}\, dW_D^A$. The same for B group, they replace parameters by the appropriate substitutions to find the equation satisfied by $\widehat{f_B}$.
- Beliefs heterogeneity: let g be the difference between group A beliefs and group B beliefs. Group A thinks that the beliefs heterogeneity is $g^A = \widehat{f_B}$ - $\widehat{f_A}$ and group B thinks that the beliefs heterogeneity is $g^B = \widehat{f_A}$ - $\widehat{f_B}$. Both processes g^A and g^B are mean reverting to their long run trend which is equal to zero since in equilibrium group A and group B have the same beliefs. The variance $\sigma_g = \sqrt{2}\, \phi\, \sigma_f$
- Excess volatility in stock prices: P_t^A and P_t^B are respectively the price that group A and group B willing to receive from selling the asset, it reflects their fundamental valuation based on information they have at time t and their perspective about capital gains. For each group the price is defined as maximization of their expectations, it is sum of two terms, the first one is accumulation of dividends present value to perceive from holding the asset between time t and transaction time t+τ, the second term is a present value of the price of the asset at transaction time minus transaction costs. $P_t^{A/B} = \max_{\tau \geq 0} E_t^{A/B}\left[\int_t^{t+\tau} e^{-r(s-t)}\, dD_S\ +\right.$

$e^{-r\,\tau}\,(\overline{P_t^{A/B}} \;-\; c)\Big]$. It is like agents of a group try to maximize the price they will receive from selling the asset to the other group by looking for the right moment $t+\tau$ that maximizes the expected present value of profit from perceiving dividends and the expected present value that the other agents are ready to pay minus the transaction cost that the seller pays. After substituting dD_S by its formula, they get $P_t^{A/B} = \max_{\tau\geq 0} E_t^{A/B}\left[\int_t^{t+\tau} e^{-r(s-t)}\,(\bar{f} + \;e^{-\lambda(s-t)}\;(\overline{\hat{f}_t^{\frac{A}{B}} - \bar{f}})ds + \;\; e^{-r\,\tau}\,(\overline{P_t^{\frac{A}{B}}} \;-\; c)\right]$,

and as a solution maximizing the formula above they find $P_t^{A/B} = \frac{\bar{f}}{r} + \frac{\hat{f}_t^{A/B} - \bar{f}}{r+\lambda} + q(g_t^{A/B})$ where the first two terms are the present value of future dividends expected by groups A or B in basis of their beliefs whereas the third term is function of beliefs heterogeneity $g^A = \widehat{f_B}$ - $\widehat{f_A}$ or $g^B = \widehat{f_A}$ - $\widehat{f_B}$ at time t, it depends on if agents A are the buyer or the seller. Finally they define the volatility of asset price as sum of the volatility of fundamental value and the volatility of option value which is $q(g_t^{A/B})$. From this finding they deduce that the volatility of stock prices exceeds the volatility of fundamental value and the excess is variance of $q(g_t^{A/B})$ which depends on overconfidence ϕ, the variance of fundamental value σ_f, the variance of agents beliefs σ_S and mean reversion speed parameter λ

C. Is this model consistent with our study

From the above model and analysis, we conclude that indeed overconfidence and beliefs heterogeneity contribute in creating excess volatility in stock prices relatively to their fundamental values. When overconfidence ϕ increases, volatility of heterogeneous beliefs increases and hence the excess volatility becomes greater. The model supposes a stochastic dividends process, considering only properties of companies without giving place to Knightian uncertainty or caring about it since investors are interested in dividends process by considering fluctuations inside companies. Talking about overconfidence, we consider it rational behavior, investors trust their beliefs and rely on their own forecasts to take investment decisions, in all steps of the model, investors are acting rationally, they use the same dividends process suggested, they attempt to assess unknowns in the model on their owns using rational information and they approximate stochastic processes regarding their properties; by these specifications, the model removes the effect of investors behavior irrationality. Also, for the model, we don't consider iterations of the process in a way to give investors the opportunity to learn from the previous information in the previous trading sessions, hence the effect of learning

factor is also removed and the model test only the effect of overconfidence factor regardless the others.

CHAPTER 4 – STOCK PRICES EXCESS VOLATILITY AND EQUITY RISK PREMIUM

In the previous studies, the objective of researchers was in the first time to prove the existence of excess volatility in stock market, they used different methods and applied them either on stock prices index or in a cross section of all stocks in the market. Whereas in the second stage, researchers moved to find and discuss explanations of excess dispersion of stock prices, they used some models to show the contribution of factors that are responsible for Excess volatility. Now, the big interest became how to conceive new asset pricing models where excess volatility takes place as common risk factor in Equity Risk Premium model.

As our interest is to include excess volatility factor in equity risk premium models as a common risk factor, I started this chapter by reminding some properties and characteristics of equity risk premium and reviewing the most important findings in equity risk premium models and the general methodology adopted previously by researchers to add a new factor in equity risk premium model, this methodology remains the most common one. In the second section, I will present the model that I suggest to take into consideration stock prices excess volatility as common risk factor in equity risk premium model, I present my work and findings that I report regarding excess dispersion factor and equity risk premium model and at the last section I report the model that was introduced by Wang and Mac in their research papers in 2014, this model was the first and the only one which proposes how to integrate excess volatility factor in equity risk premium models, it tackled the most important previous findings in excess volatility modelling and asset pricing models.

I. EQUITY RISK PREMIUM PROPERTIES AND MODELLING

A. Equity risk premium properties

In finance, as maybe in all domains, risk matters as much as return, this couple of parameters is the basis and the criterion in investment decisions and choices. Investors are required to care their investment strategies and make sure that all risky investments are as remunerated as the risk they generate. In stock market, risk notion takes more place than in other financial products since stocks are the most volatile and the riskiest product. Thus, investors are involved to expect their returns or more precisely to require returns that they think they are necessary to compensate the risk they are exposed to through the risky assets. The expected returns or rather the required returns reflect the set of risks that investors are worried about and they are conscious of. They condition the expected returns of stocks by all risk factors in the market; they set returns on any investment as the sum of risk free rate and risk premium to compensate for the set of risks they face. Investors become very conscious to assume their position with respect to risk, the most common and frequent way to

account risk in their strategies is to find measure to assess and quantify risk and then impose remuneration for it via equity risk premiums. Hence, investors are concerned by developing equity risk premium models that include all forms and factors of risk and require returns that must remunerate the risky assets.

These specifications covering the couple risk-return give priority to equity risk premium modelling issue; it occupies large space in financial modelling, it was and it remains very important subject since investors are interested to make clear the risk they face versus the return they ask for and since the market is always in progress; new factors may appear in future as well as the effect of others may disappear if the agents in the market adjust their strategies according to these factors. Thus the equity risk premium will always be the bottom of subjects in the market. Our main interest in this study as in the real market is to estimate equity risk premium regarding all common risk factors. The approach consists on looking for determinants and factors of common risk those impact investors' choices and investigating the premiums to require for being exposed to such risk. Every risky investment requires compensation so that investors can feel in a certain level secure towards risk of their investments. Consequently, it is very important to explore determinants and factors that take part in Equity risk premium. Different factors have been tackled in previous studies within the framework of equity risk premium modelling, market risk, size risk and book to market ratio factor, in our study we focus on adding a new common risk factor which is stock prices excess volatility.

B. Review of equity risk premium models

Equity risk premium is the key to accept the coexistence of risk-return, investors accept to invest in risky assets because they know they will be compensated for risk and they will receive the price of the risk. The higher the risk is the higher the equity risk premium. Both investors and researchers are convinced that they must be able to assess all risks surrounding stocks and assign the appropriate equity risk premium for each one. To do that, researchers have been discussing in their research papers all kind of risks they affect returns, how to assess them and find proxies for them as well as integrate them in an appropriate way in equity risk premium models. Specific risk in stocks depends on each stock and in general it can be removed from portfolios that investors invest on, but for common risk is not, hence in equity risk premium models, only common risk is discussed. Researchers consider multi-index model as the most appropriate model to tackle equity risk premium and common risk factors, it consists on expecting excess returns as the sum of parameters representing the common risk factors and multiplied by returns of investment portfolios that are constructed in basis of these factors. The first form of this model was the CAPM; it represents excess returns or equity risk premiums by considering market risk proxied by beta as the first common risk factor and multiplied by the difference between market index returns and risk free asset returns.

Then several other models have been suggested to add other common risk factors to market risk. In 1992 Fama and French proposed a model with three factors, market risk, firm size risk proxied by market capitalization and growth risk proxied by book to market ratio. In their research paper they proposed a methodology that remained used by all the following models dealt with equity risk premiums. After this model, different researchers attempted to add other factors as momentum factor, information factor, liquidity factor but Fama and French model remains the most frequently used. In the following, I set all steps of the methodology used by Fama and French model in their first research paper about equity risk premium model and common risk factors:

- Before talking about adding a new factor in equity risk premium model, it is necessary to observe a new factor risk and its effects on excess returns in the market, to explore the relation return-risk and show the evidence of this relation. Fama and French observed that the smaller the firm is the riskier the investment, so investors must require higher returns for smaller firms. They also remarked the effect of another factor which is growth factor, they reported that the slower the growth of a firm the riskier the investment and hence investors must require high returns for slow growth firms.
- After fixing the new common risk factors to add in equity risk premium model, it is necessary to find proxies for the new factors, Fama and French proxied firm size factor by market capitalization and growth factor by Book to market ratio, in basis of their observations a negative correlation must be between firm size and excess returns from one hand and growth and equity risk premium from another hand.
- Their observations and logic reasoning were only to show the evident relation between the new factors and excess returns, but they went further to prove empirically these correlations. They collected historical data from the market about all stocks in US market, their daily or monthly prices, accounting data (book and market values) and the necessary information to compute betas of all stocks in the period of their study.
- For firm size factor: for each year they pick market capitalization of all stocks, they rank stocks in ascending way according to their market value and they split stocks in deciles to build ten equally weighted portfolios ranked according to their market values, this building portfolio strategy is repeated every year as trading strategy. Fama and French computed monthly excess returns of the ten portfolios, compute their averages over years and prove the negative correlation between size and excess returns since the first portfolio with the lowest market value requires the highest excess returns and the last portfolio with the highest market value requires the lowest excess returns.

- For growth factor: for each year Fama and French pick book and market values of all stocks, they compute book to market ratios, rank stocks in descending way according to their book to market ratio and split stocks in deciles to build ten equally weighted portfolios ranked according to their growth, this building portfolio strategy is repeated every year as trading strategy. Fama and French compute monthly excess returns of the ten portfolios, compute their averages over years and prove the negative correlation between growth and excess returns since the first portfolio with the highest book to market ratio and eventually with the slowest growth requires the highest excess returns and the last portfolio with the lowest book to market ratio and the fastest growth requires the lowest excess returns.
- After proving the correlations between the new factors and stocks excess returns, Fama and French moved to build their model, it is cross section regression of excess returns on equity risk premium factors namely market risk, firm size and growth factors which are the independent variables in regression. Fama and French build the dependent and the independent variables series' over time of all stocks then they do regressions for all individual stocks and compute averages of regression coefficients over stocks.
- The final step in Fama and French methodology is to test the statistical significance of regression coefficients in the equity risk premium model, they get in the last step the following model:

$$R_{i,t} - R_{f,t} = \alpha_1 * \beta_{i,t} + \alpha_2 * \ln(FirmSize_{i,t}) + \alpha_3 * \ln(BTM_{i,t})$$

Other factors may be added in this model according to the same methodology we explored so far.

II. EXCESS VOLATILITY COMMON RISK FACTOR IN EQUITY RISK PREMIUM MODEL

A. Stock prices mean reverting towards their fundamental values

Excess volatility refers to excess dispersion of market stock prices relatively to dividends present value, in other words as Shiller defined it, excess volatility is the variance which is not explained by volatility of prices in dividends present value model. The struggle to measure excess volatility is that P_t^* is not observable in the market since it is the sum of present values of dividends in the whole stock life. In market microstructure, stocks are characterized by prices and returns that are mean reverting, so we can consider that P_t^* as long run trend of P_t and P_t is mean reverting towards P_t^*, prices are always mean reverting to their fundamental values even in extreme cases like in bubbles and crashes. Stock prices diverge from fundamental values by the action of feedback traders, but eventually these actions are reversed when the market realizes that share price has diverged too far away from the fundamental value and then they revert back to their fundamental values.

All these specifications push us to admit that market stock prices P_t are mean reverting towards their fundamental values P_t^*. Our findings are strongly consistent with what Raymond Chiang, Peter Liu, John Okunev find in their research papers. In 1995 Raymond Chiang, Peter Liu, John Okunev tackled the issue of mean reversion process of stock prices towards their fundamental values and they mentioned that a lot of researchers confirmed the same finding namely DeBondt and Thaler (1985, 1987), Campbell and Shiller (1988), Poterba and Summers (1986), Lo and MacKinlay (1988), Fama and French (1988a,b), Fama (1990), Cecchetti et al. (1990), Campbell (1991), Ferson and Harvey (1991), McQueen and Thodey (1991) and Campbell and Kyle (1993). They propose that stock prices represent evidence of mean reversion towards their fundamental values.

The market value of the asset is modelled as a mean reverting Ornstein Uhlenbeck process towards the fundamental value. The process of mean reversion that Raymond Chiang, Peter Liu, John Okunev proposed consists on: $dP_t = \lambda * (P_t^* - P_t)\, dt + \sigma\, dZ_t$ where P_t and P_t^* are respectively market stock price and fundamental value at time t, λ is mean reversion speed which must be positive, σ is variance of dP_t stock price change and Z_t is standard wiener process with zero mean and unit variance. The idea behind this process is when market stock prices P_t go up away from their fundamental values P_t^*, the difference $(P_t^* - P_t)$ becomes negative and certain strength pulls down market stock prices P_t towards P_t^* at λ speed that's why the change in stock prices dP_t must be negative and its volatility is σ. And it is analogous to the case where market stock prices P_t go down away from their fundamental values P_t^*. The difference $(P_t^* - P_t)$ becomes positive and certain strength pulls up market stock prices P_t towards P_t^* at λ speed that's why the change in stock prices dP_t must be positive and its volatility is σ. Before using this process Raymond Chiang, Peter Liu, John Okunev propose empirical method to check if indeed the market data satisfies this process. They make the proposition that fundamental value P_t^* follows the following process $\frac{dP_t^*}{P_t^*} = \mu\, dt + \delta\, dW_t$ which is a geometric wiener process. Then they substitute this process in mean reversion process of market stock prices towards their fundamental values to get at the end the following equation $\Delta P_t = \gamma_1 * P_t + \gamma_2 * \Delta D_t + \gamma_3 * D_t + e_t$ where ΔP_t are annual changes in market stock prices, P_t are annual market stock prices, ΔD_t are annual changes in dividends, D_t are annual dividends. γ_1, γ_2 and γ_3 are regression coefficients and γ_1 is equal to $(e^{-\lambda * \Delta t} - 1)$ and it serves to find the value of mean reversion speed. To make sure that the data satisfies the mean reversion process of stock prices towards their fundamental values, we gather annual observations of the dependent and the independent variables from the market for a long period and we regress the dependent variable on the independent variables. The mean reversion process of stock prices towards their fundamental values is valid by the data when all regression coefficients γ_1, γ_2 and γ_3 are statistically significant, the value of mean reversion speed λ is significantly positive, because if

it is null the process is random walk rather than mean reversion and if it is negative the stock prices go far away from their fundamental values without reverting and the third thing to check is the R-squared to assess the explanatory power of this model. Raymond Chiang, Peter Liu, John Okunev propose mean reversion process of market stock prices towards their fundamental values and suggest an empirical method to verify if the market data satisfies the process and hence if we can use this process to model stock prices. We will use in our excess volatility proxy the mean reversion process of stock prices but of course we will before verify if the data we pick from the market satisfies this process.

B. Excess Volatility: Common Risk factor

The first question we must ask before dealing with a factor as common risk factor is why we have to consider it as a risk and why investors are concerned by this risk factor. Excess volatility is excess dispersion in stock prices; market stock prices are moving and fluctuating around their fundamental values more than they should be; this over-dispersion makes prices less predictable and hence riskier. When stock prices go largely up or largely down from their fundamental values, investors' losses become more frequent and large because of these over-valuations and under-valuations. Excess volatility adds more uncertainty in stock prices and so more risk that investors would be exposed to; they ask for risk premiums to compensate excess volatility risk and manage this risk.

As we have seen in the previous sections, excess volatility is result of set of factors, namely Knightian uncertainty which is result of uncertain economic, political and government policies, investors behavior, learning and overconfidence which are rational behavior and optimism, pessimism, speculative trading, and noise trading which are irrational behavior, all these factors are common between all stocks. Excess volatility infects all stocks in the market since it depends on movements in the market, on investors' behavior and global uncertainty. All stocks are infected by these components and hence excess volatility is a common risk factor. As the risk market, firm size and growth factors in equity risk premium model, excess volatility should also be treated as common risk factor in risk-adjusted returns model.

The previous aspects of Excess volatility issue tackled the existence of excess volatility in stock market and factors that are responsible for stock prices excess dispersion. They have been conducted without having need to measure excess volatility and so it was not necessary to find a proxy for excess volatility factor but the excess volatility as common risk factor involves investors to take into consideration excess volatility risk in their expectations of daily, monthly and annual returns and this requires that investors must be able to quantify excess volatility risk and integrate it in equity risk premium models.

C. Excess Volatility Proxy

By definition excess volatility is the volatility of market stock prices which is not explained by the model of dividends present value. This term of unexplained excess volatility remind us the term of the fraction of unexplained variance by the model which appears in the R squared formula. Formula of R squared $R^2 = 1 - \frac{Unexplained\ variance\ by\ the\ model}{Total\ variance} = 1 - \frac{Var\ (y - \tilde{y})}{Var\ (y - \bar{y})}$ Where $\bar{y}$ is mean of real y and $\tilde{y}$ is the model dedicated to expect y, from this remark we can say that Excess volatility measure must be something like $EV = VAR(P - P^*)$.

From all the remarks we discussed in the previous section and the findings in Raymond Chiang, Peter Liu, John Okunev research paper in 1995, we admit that market stock prices are mean reverting towards their fundamental values; after making sure, over the empirical study we will do in the following section, that data from the market satisfies this process we can use it in the remainder of this chapter. Mean reversion process consists on that market stock prices revert towards their fundamental values according to the following process $dP_t = \lambda * (P_t^* - P_t)\, dt + \sigma\, dZ_t$. In exchange market, investors deal with daily prices rather than instantaneous prices. Thus, when they do their analysis and explore their forecasts, they have been interested in observing daily movements of prices and risk factors that may impact daily changes in prices between today and tomorrow. Moreover, the most relevant data in the market is daily prices data. Because of all these specifications, we would define excess volatility factor proxy as excess volatility in daily prices relatively to their fundamental values and the same measure would be used to express excess volatility risk in monthly and annual returns. We integrate the above formula of mean reversion process in one day between t and $t + \Delta t$ since Δt represents one day, we got

$\int_t^{t+\Delta t} dP_t = \int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt + \int_t^{t+\Delta t} \sigma\, dZ_t$. We set the integrals $\int_t^{t+\Delta t} dP_t = \Delta P_t$ and $\int_t^{t+\Delta t} \sigma\, dZ_t = \sigma \int_t^{t+\Delta t} dZ_t = \sigma * \Delta Z_t$ we substitute and we got the formula below

$\Delta P_t = \int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt + \sigma * \Delta Z_t$. We saw the evidence from excess volatility as excess dispersion of stock prices relatively to their fundamental values or also the variance of market stock prices P_t which is not explained by the model P_t^* and we said that from R-squared evidence we can see that excess volatility is something similar to $EV = VAR(P - P^*)$. From these both specifications of mean reversion process of stock prices towards their fundamental values and the evidence from excess volatility definition we set that excess volatility might be proxied by

$EV = Var\ \left(\int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt\right)$. Since as we mentioned above the excess volatility is linked directly to daily prices, we set in the formula integral of instantaneously prices on one day. Mean reversion speed λ takes its place in excess volatility formula exactly as it should do, because when mean reversion speed is high, the investors overreaction is high and the excess volatility is high

too, but when the mean reversion speed is low, the investors overreaction is low and the excess volatility is low too, that is why we should consider mean reversion speed in excess volatility formula. Now from the above analysis we got the formula of stock prices excess volatility as $EV = Var\left(\int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt\right)$.

We go back to mean reversion property of stock prices $\Delta P_t = \int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt + \sigma * \Delta Z_t$. Excess volatility is defined by $EV = Var\left(\int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt\right)$, we then apply variance operator on

$\Delta P_t = \int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt + \sigma * \Delta Z_t$ to get $Var(\Delta P_t) = Var\left(\int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt + \sigma * \Delta Z_t\right)$

Then

$$Var(\Delta P_t) = Var\left(\int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt\right) + \mathrm{Var}\,(\sigma * \Delta Z_t) + 2\, Cov\,(\int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt\ ;\ \sigma * \Delta Z_t)$$

In the formula just above, we emphasis the excess volatility proxy: $Var(\Delta P_t) = EV + \sigma^2\, \mathrm{Var}\,(\Delta Z_t) + 2\, Cov\,(\int_t^{t+\Delta t} \lambda * (P_t^* - P_t)\, dt\ ;\ \sigma * \Delta Z_t)$

From stochastic calculus properties we have the variance of standard wiener process change is $\mathrm{Var}\,(\Delta Z_t) = Var\,(Z_{t+\Delta t} - Z_t) = t + \Delta t - t = \Delta t$

$$Var(\Delta P_t) = EV + \sigma^2\, \Delta t + 2\, Cov\,(\int_t^{t+\Delta t} \lambda * P_t^*\, dt\ ;\ \sigma * \Delta Z_t) - 2\, Cov\,(\int_t^{t+\Delta t} \lambda * P_t\, dt\ ;\ \sigma * \Delta Z_t)$$

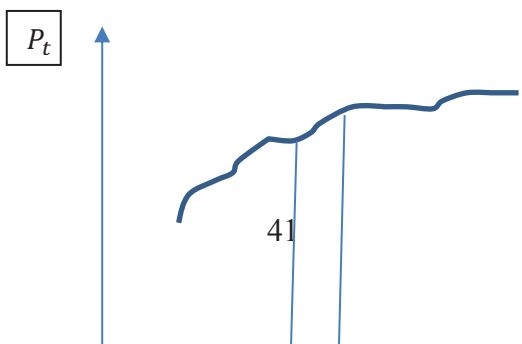
And we know that the fundamental values of stock prices are uncorrelated to market stock prices noise which means that $Cov\left(\int_t^{t+\Delta t} \lambda * P_t^*\, dt\ ;\ \sigma * \Delta Z_t\right) = 0$

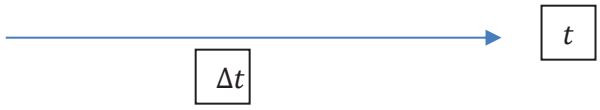
$$Var(\Delta P_t) = EV + \sigma^2\, \Delta t - 2\, Cov\,(\int_t^{t+\Delta t} \lambda * P_t\, dt\ ;\ \sigma * \Delta Z_t)$$

As we mentioned above, the mean reversion process of stock prices set the property that σ is the volatility of ΔP_t by unit of time, since the variance $Var(\Delta P_t)$ is variance of changes in daily prices and $\sigma^2\, \Delta t$ is variance in daily period Δt, this means that $Var(\Delta P_t) = \sigma^2\, \Delta t$, we got then the following formula for stock prices excess volatility $EV = 2\, Cov\,(\int_t^{t+\Delta t} \lambda * P_t\, dt\ ;\ \sigma * \Delta Z_t)$

Then $EV = 2\, \lambda\, \sigma\, Cov\,(\int_t^{t+\Delta t} P_t\, dt\ ;\ \Delta Z_t)$, to simplify we consider that the change in daily price is as below in the graph

Figure 2 : Prices Movements in one day





We consider the change in daily stock prices in one day Δt as approximation in the graph above and we set $\int_t^{t+\Delta t} P_t\, dt = P_{t+\Delta t} * \int_t^{t+\Delta t} dt = \Delta t\, P_{t+\Delta t} = \Delta t\, P_t + \Delta t\, \Delta P_t$, this approximation is consistent with the fact that investors consider in their computations only close price at the end of the day which is $P_{t+\Delta t}$. We substitute in excess volatility formula to find:

$$EV = 2\,\lambda\,\sigma\, Cov\,(\Delta t\, P_t + \Delta t\, \Delta P_t\ ;\ \Delta Z_t)$$

$$EV = 2\,\lambda\,\sigma\,\Delta t\, Cov\,(P_t + \Delta P_t\ ;\ \Delta Z_t) = 2\,\lambda\,\sigma\,\Delta t\,[Cov\,(P_t\ ;\ \Delta Z_t) + Cov\,(\Delta P_t\ ;\ \Delta Z_t)]$$

$$EV = 2\,\lambda\,\sigma\,\Delta t\,\Big[\sqrt{Var(P_t)}\sqrt{Var(\Delta Z_t)}\, Corr\,(P_t\ ;\ \Delta Z_t) + \sqrt{Var(\Delta P_t)}\sqrt{Var(\Delta Z_t)}\, Corr\,(\Delta P_t\ ;\ \Delta Z_t)\Big]$$

$$EV = 2\,\lambda\,\sigma\,\Delta t\,\Big[\sqrt{Var(P_t)}\sqrt{\Delta t}\, Corr\,(P_t\ ;\ \Delta Z_t) + \sqrt{Var(\Delta P_t)}\sqrt{\Delta t}\, Corr\,(\Delta P_t\ ;\ \Delta Z_t)\Big]$$

$$EV = 2\,\lambda\,\sigma\,\Delta t\,\sqrt{\Delta t}\,\Big[\sqrt{Var(P_t)}\, Corr\,(P_t\ ;\ \Delta Z_t) + \sqrt{Var(\Delta P_t)}\, Corr\,(\Delta P_t\ ;\ \Delta Z_t)\Big]$$

We know that $\sigma\sqrt{\Delta t} = \sqrt{Var(\Delta P_t)}$

$$EV = 2\,\lambda\,\Delta t\,\sqrt{Var(\Delta P_t)}\,\Big[\sqrt{Var(P_t)}\, Corr\,(P_t\ ;\ \Delta Z_t) + \sqrt{Var(\Delta P_t)}\, Corr\,(\Delta P_t\ ;\ \Delta Z_t)\Big]$$

We need now to specify $Corr\,(P_t\ ;\ \Delta Z_t)$

$$Corr\,(P_t\ ;\ \Delta Z_t) = \frac{E(P_t * \Delta Z_t) - E(P_t) * E(\Delta Z_t)}{\sigma_{P_t} * \sigma_{\Delta Z_t}}$$

$$Corr\,(P_t\ ;\ \Delta Z_t) = \frac{E((P_t - P_{t+\Delta t} + P_{t+\Delta t}) * \Delta Z_t) - E(P_t - P_{t+\Delta t} + P_{t+\Delta t}) * E(\Delta Z_t)}{\sigma_{P_t} * \sigma_{\Delta Z_t}}$$

$$Corr\,(P_t\ ;\ \Delta Z_t) = \frac{E((P_t - P_{t+\Delta t}) * \Delta Z_t) - E(P_t - P_{t+\Delta t}) * E(\Delta Z_t)}{\sigma_{P_t} * \sigma_{\Delta Z_t}} + \frac{E(P_{t+\Delta t} * \Delta Z_t) - E(P_{t+\Delta t}) * E(\Delta Z_t)}{\sigma_{P_t} * \sigma_{\Delta Z_t}}$$

$$Corr\,(P_t\ ;\ \Delta Z_t) = -\frac{E(\Delta P_t * \Delta Z_t) - E(\Delta P_t) * E(\Delta Z_t)}{\sigma_{P_t} * \sigma_{\Delta Z_t}} + \frac{E(P_{t+\Delta t} * \Delta Z_t) - E(P_{t+\Delta t}) * E(\Delta Z_t)}{\sigma_{P_t} * \sigma_{\Delta Z_t}}$$

$$Corr\,(P_t\ ;\ \Delta Z_t) = -\,Corr\,(\Delta P_t\ ;\ \Delta Z_t)\frac{\sigma_{\Delta P_t}}{\sigma_{P_t}} + Corr\,(P_{t+\Delta t}\ ;\ \Delta Z_t)\frac{\sigma_{P_{t+\Delta t}}}{\sigma_{P_t}}$$

Since ΔZ_t is noise of $\Delta P_t = P_{t+\Delta t} - P_t$ and it is positively correlated to it, this means that when ΔZ_t increases ΔP_t does and so $P_{t+\Delta t}$ increases and $P_t$decreases and the reverse if ΔZ_t decreases. So can approximate $Corr\,(P_{t+\Delta t}\ ;\ \Delta Z_t) = -Corr\,(P_t\ ;\ \Delta Z_t)$, we get then

$$Corr\,(P_t\ ;\ \Delta Z_t) = -\,Corr\,(\Delta P_t\ ;\ \Delta Z_t)\frac{\sigma_{\Delta P_t}}{\sigma_{P_t}} - Corr\,(P_t\ ;\ \Delta Z_t)\frac{\sigma_{P_{t+\Delta t}}}{\sigma_{P_t}}$$

$Corr\ (\ P_t\ ;\ \ \Delta Z_t) = \frac{-\frac{\sigma_{\Delta P_t}}{\sigma_{P_t}}}{1+\frac{\sigma_{P_{t+\Delta t}}}{\sigma_{P_t}}}\ Corr\ (\ \Delta\ P_t\ ;\ \ \Delta Z_t)$ And we know that

$\sigma_{P_{t+\Delta t}}{}^2 = \sigma_{P_t+\Delta P_t}{}^2 = \sigma_{P_t}{}^2 + \sigma_{\Delta P_t}{}^2 + 2\ \sigma_{\Delta P_t}\sigma_{P_t} Corr\ (\ P_t\ ;\ \ \Delta P_t)$

We substitute all results in Excess volatility formula and we get:

$$EV = 2\lambda\Delta t\sqrt{Var(\Delta P_t)}Corr\ (\ \Delta P_t;\Delta Z_t)\left[\sqrt{Var(P_t)}\ \frac{-\sigma_{\Delta P_t}}{\sigma_{P_t}+\sqrt{\sigma_{P_t}{}^2+\sigma_{\Delta P_t}{}^2+2\ \sigma_{\Delta P_t}\sigma_{P_t}Corr(P_t;\Delta P_t)}} + \sqrt{Var(\Delta P_t)}\right]$$

As ΔZ_t represents all other factors that take action on ΔP_t in the market, other than excess volatility factor, we considered in the mean reversion model the ΔZ_t is all ΔP_t factors and excess volatility factor is noise of ΔP_t , so we approximate $Corr\ (\ \Delta P_t;\Delta Z_t) = 1$

$$EV = 2\lambda\Delta t\sqrt{Var(\Delta P_t)}\left[\sqrt{Var(P_t)}\ \frac{-\sigma_{\Delta P_t}}{\sigma_{P_t}+\sqrt{\sigma_{P_t}{}^2+\sigma_{\Delta P_t}{}^2+2\ \sigma_{\Delta P_t}\sigma_{P_t}Corr(P_t;\Delta P_t)}} + \sqrt{Var(\Delta P_t)}\right]$$

$$EV = 2\ \lambda\ \Delta t\ \sigma_{\Delta P_t}{}^2\left[\frac{-\sigma_{P_t}}{\sigma_{P_t}+\sqrt{\sigma_{P_t}{}^2+\sigma_{\Delta P_t}{}^2+2\ \sigma_{\Delta P_t}\sigma_{P_t}Corr(P_t;\Delta P_t)}} + 1\right]$$

All parameters and components in excess volatility formula above are observable in the market; we compute all variances and standard deviations of daily variables in basis of one year historical data. We observe also that excess volatility proxy is always positive, this is consistent with the fact that market stock prices are always more volatile relatively to their fundamental values.

D. Excess Volatility Proxy and excess volatility factors

We must verify if factors that we discussed above their cause effect relation with excess volatility are consistent with the proxy that we chose to measure excess volatility or in the right words if the proxy is congruent with what have been said about these factors. In irrational behavior of investors factor we discovered how this latter contributes in creating excess dispersion in stock prices, we deduced that when investors expect high earnings, they consequently expect high prices, the prices are highly set in the market, they go up away from their fundamental values but when investors get aware of the real values they set low prices in the market, and because of their irrational behavior prices keep going down away from their fundamental values and so we can obviously see how prices

are fluctuating around their fundamental values in mean reversion process. The effect of investors' irrationality is somehow taken into consideration in excess volatility proxy.

By exploring overconfidence factor, we knew that investors beliefs about dividends process are mean reverting towards their long run trends and since market prices reflect investors' beliefs, we can obviously make the link between investors' beliefs process and market stock prices process to offset the mean reversion property from investors' behavior to market stock prices. This is consistent with our objective to include overconfidence effect in excess volatility proxy.

Both P_t^* and P_t are random variables and their variances cannot be computed since we cannot get their own observations, as rational behavior investors use learning methods to compute variances of these variables via historical data, the proxy we attributed to excess volatility reflects exactly what investors do in real market and it represents also the effect of learning factor in creating excess dispersion via the use of historical data.

The distributions and probabilities that investors use in real market are subjective distributions; in presence of Knightian uncertainty investors do the same, they use their own distributions which contribute in a way in creating excess volatility in stock prices. As in our study we supposed that investors consider only close prices, they don't consider intraday volatility. We can see this assumption as a way to represent the effect of subjective distributions used by investors and by the way to take into consideration Knightian uncertainty effect in excess volatility proxy and that's why we supposed in the integral above that price is set to $P_{t+\Delta t}$ between t and $t + \Delta t$.

E. Excess Volatility factor in Equity Risk Premium model

We set the methodology we follow to conceive the new equity risk premium model which takes into consideration excess volatility factor then we move to discuss some empirical results. We can use exactly the same model that Fama and French proposed in their research paper in 1992 in which they consider the betas, market capitalization and book to market value are the independent variables and regress monthly returns on these variables to find regression coefficients or we can also do regressions otherwise which means regress returns on returns. In our study we use regress returns on returns methodology, this consists on building investment portfolios in basis of the factors beta, firm size, growth and excess volatility then we regress monthly returns of stocks on monthly returns of the investment portfolios to get values of regression coefficients for all individual stocks then compute their averages over stocks. These regression coefficients represent somehow the averages over the whole sample of factors betas, market capitalization, book to market value and excess volatility. The steps we should follow to integrate excess volatility in the new model are set as follows:

- We choose US market as example to test this model on; we gather accounting and financial data about all stocks listed in US market, we fix period to conduct our study in and make sure that all stocks in our sample have the necessary data over the whole period.
- First of all we need to check the correlation between excess returns and excess volatility factor, we can do that for all individual stocks in US market or only for US market index. We compute monthly excess returns as the difference between monthly returns of individual stocks or market index and monthly risk free rate as interest rates of 1-month treasury bills. We then compute monthly values of excess volatility proxy for each individual stock or only for market index. For each month we use 1 previous year historical data to estimate the value of excess volatility as we explained above in the formula of excess volatility proxy. We compute correlation coefficient between monthly returns of market index and monthly values of excess volatility associated to market index, the correlation coefficient must be positive since investors ask for high returns to compensate the risk of high excess volatility. If stocks have high excess volatility, investors are not ready to pay too much for this stock, hence prices in the market are low, this is what we call the price of risk, then after while prices increase since investors become aware of their fundamental values and in this way stocks with high excess volatility give high returns.
- After making sure that excess returns are positively correlated to excess volatility we move to do regressions according to the following tips.
- For all individual stocks, for each year t we pick accounting data, book value and market value that are set in their financial statements at the end of each fiscal year (t-1) to be used for the year t, both values are used to compute book to market ratios. Each year we rank assets in descending way according to book to market ratios and we set them into deciles. We get each year ten portfolios equally weighted, the first one has the highest book to market ratio and the tenth one has the lowest book to market ratio. These two portfolios are the investment portfolios based on book to market ratio, they are hold for one year then renewed the next year. We compute monthly returns of these two portfolios, the difference between monthly returns of the first portfolio and monthly returns of the tenth portfolios is considered the independent variable which represents book to market ratio factor, this choice is obvious since the portfolio with the highest book to market ratio which means with the slowest growth requires higher returns and the portfolio with the lowest book to market ratio which means with the fastest growth requires lower returns. We do the same building strategy each year and we get series of book to market based monthly returns.

- We do exactly the same trading strategies for size firm factor. We pick market capitalization in each year t for all stocks, we rank stocks in ascending way according to their market value, and we classify them in ten deciles. We get ten market capitalization based portfolios, the first portfolio with the lowest market value is supposed to have the highest returns and the last portfolio with the highest market capitalization is supposed to have the lowest required returns. We repeat the same operations each year in a way to get each year market capitalization based investment portfolios; these portfolios are hold for one year then renewed the next year. We compute monthly returns of the first and the tenth portfolios, the independent variable associated to market capitalization factor is the difference between monthly returns of the lowest market value portfolio and monthly returns of the highest market value portfolio, this choice is obvious since the portfolio with the lowest market value which means with the lowest size requires higher returns and the portfolio with the highest market value which means with the highest size requires lower returns. In this way we build series of market capitalization independent variable.
- For the independent variable associated to the third factor which is beta it is always chosen as the market risk premium which is the difference between market returns and risk free rate. For this we need to pick monthly returns of US market index and risk free rate as interest rate of 1-month US treasury bills and compute monthly series of the difference between market returns and risk free rate. In this way the series for the third independent variable representing beta factor are complete.
- Now, we have the series of the independent variables, beta, firm size and book to market ratio, we then do regressions for all individual stocks to test Fama and French model before testing our model with the new factor. We regress returns on returns; monthly excess returns of all individual stocks as the dependent variable on the Fama and French model factors as we explained so far $R_{i,t} - R_{f,t} = \alpha_1 * \left(R_{m,t} - R_{f,t} \right) + \alpha_2 * SMB_t + \alpha_3 * HML_t$. After we get regression coefficients for all individual stocks, we compute averages of these regression coefficients over all individual stocks; we analyze statistical significance of regression coefficients and statistical significance of intercept to check if Fama and French model fits the data, in this case we must find that all regression coefficients are statistically significant.
- Then we move to verify the explanatory power of the new model with excess volatility factor $R_{i,t} - R_{f,t} = \alpha_1 * \left(R_{m,t} - R_{f,t} \right) + \alpha_2 * SMB_t + \alpha_3 * HML_t + \alpha_4 * EV_t$, after regressing returns on returns within the framework of Fama and French model, we add the new factor associated to excess volatility. After making sure that excess volatility is positively

correlated to excess returns as for the previous factors, we build investment portfolios based on excess volatility factor using the same trading strategy. Each year we compute for all individual stocks excess volatility factor proxy based on 1 previous year historical data, we rank stocks in descending way according to excess volatility factor and we classify them in deciles. We get ten portfolios, the first portfolio has the highest excess volatility and it must require the highest returns since investors ask for high returns as compensation to be exposed to excess volatility risk, the last portfolio has the lowest excess volatility and it must require the lowest returns. These portfolios are hold for one year and renewed the next year, the independent variable associated to excess volatility factor is the difference between monthly returns of the highest excess volatility portfolio and monthly returns of the lowest excess volatility portfolio. The series for excess volatility independent variable are complete.

- We regress returns on returns as the same way we did in Fama and French model but this time we add the new factor which is excess volatility factor. We do regressions for all individual stocks to test the new model with the new factor. We regress returns on returns; monthly excess returns of all individual stocks as the dependent variable on the Fama and French model factors and excess volatility factor using the independent variables series we computed. After we get regression coefficients for all individual stocks, we compute averages of these regression coefficients over all individual stocks; we analyze statistical significance of regression coefficients and statistical significance of intercept to check if the new model with excess volatility factor fits the data, in this case we must find that all regression coefficients of Fama and French factors remain statistically significant even in presence of the new factor, the regression coefficient of excess volatility factor must also be statistically significant whereas the intercept must not be statistically significant. If all these conditions are satisfied we valid the new proxy of excess volatility factor and the new equity risk premium model integrating excess volatility as common risk factor.

Empirical results

Before talking about regressions within the framework of the model, we need first of all to valid our use of the mean reversion process of stock prices towards their fundamental values, because this is the main model on which we were based to build the proxy of excess volatility: as mentioned before, the process consists on : $dP_t = \lambda * (P_t^* - P_t)\, dt + \sigma\, dZ_t$, we used exactly this equation to build our proxy, but to validate this process we use the second equation $\Delta P_t = \gamma_1 * P_t + \gamma_2 * \Delta D_t + \gamma_3 * D_t + e_t$, since in the first one, not all components are observable in the market, this is what Raymond Chiang, Peter Liu, John Okunev did in their research paper in 1995 and it is exactly

what we do to check the validity of this hypothesis. We gathered the data of US market index, annual observations of ΔP_t, P_t, ΔD_t and D_t for the same period of our study and we did regressions. In regressions we used Nominal values of ΔP_t, P_t, ΔD_t and D_t and deflated values that are adjusted to inflation. We did regressions for both. For regression with Nominal values we found the results in the table below:

Table 3 : Empirical tests of Mean Reversion Process: Nominal Values

	Coefficients	*Erreur-type*	*Statistique t*	*Probabilité*	*Limite inférieure pour seuil de confiance = 95%*	*Limite supérieure pour seuil de confiance = 95%*
Constante	0	#N/A	#N/A	#N/A	#N/A	#N/A
Annual P	-0,118947543	0,040799131	-2,915443091	0,004133102	-0,199604629	-0,038290457
Annual D	5,267320566	2,086025351	2,52505108	0,012674833	1,143391362	9,391249769
annual dD	52,54668176	5,877646425	8,940088934	1,96244E-15	40,92697747	64,16638606

All regression coefficients are significant at 95%. According to the model, regression coefficient of annual Prices P variable is exactly ($exp(-\lambda * dt) - 1$). We can so find the value of λ which is speed of stock prices mean reversion to their fundamental values per year. λ = 12.66%, we found positive value of speed, which means that yes there is mean reversion of stock prices towards their fundamental values at 12.66%. If we found null value of λ, this means that stock prices would have random walk process, if we found negative value of λ, this means that stock prices would progress away and would not revert to their fundamental values. R squared equal to 69.89%, which means that the model have 69.89% explanatory power, so we accept the model and the assumptions we made. For the second regression with the deflated values we got the results below:

Table 4 : Empirical tests of Mean Reversion Process: Deflated Values

	Coefficients	*Erreur-type*	*Statistique t*	*Probabilité*	*Limite inférieure pour seuil de confiance = 95%*	*Limite supérieure pour seuil de confiance = 95%*
Constante	0	#N/A	#N/A	#N/A	#N/A	#N/A
Annual P	-0,059493104	0,029648976	-2,006582092	0,046748067	-0,118118128	-0,000868079
Annual D	2,376787459	1,182045336	2,010741371	0,046301024	0,03952503	4,714049887
Annual dD	31,18003467	5,409149876	5,764313319	5,13233E-08	20,48450346	41,87556588

All regression coefficients are significant at 95%. According to the model, regression coefficient of annual P variable is exactly ($exp(-\lambda * dt) - 1$). We can so find the value of λ which is speed of stock prices mean reversion to their fundamental values per year. λ = 6.13% we found positive value of speed, which means that yes there is mean reversion of stock prices towards their fundamental

values at 6.13%. R squared equal to 46.22%, which means that the model have 46.22% explanatory power, so we accept the model and the assumptions we made.

If we compare from one side the explanatory power of models using nominal terms and deflated terms, we find that the first one is powerful than the second one, if we compare from another side the mean reversion speed of models using nominal terms and deflated terms, we find that mean reversion in the first one is stronger than mean reversion in the second one, hence for the remainder we should use nominal values. These results are congruent with empirical results in the research paper of Raymond Chiang, Peter Liu, John Okunev in 1995. They found also the same results.

Excess volatility must show positive correlation with excess returns, because if stocks have high excess volatility, investors are not ready to pay too much for this stock, hence prices in the market are low, this is what we call the price of risk, then after a while prices increase since investors get aware of their fundamental values and in this way stocks with high excess volatility give high returns. To verify the positive correlation between excess volatility factor and excess returns we explore that only for US market index S&P 500. Our choice is because we don't have historical data for all stocks listed in US market, normally if we could get this data we would verify the correlation coefficients between excess volatility and excess returns for all stocks and compute the average. Since we could not get the necessary data we verified that only for S&P 500 index. We computed monthly excess returns of S&P for the whole period between 1951 and 2016; we compute monthly values of excess volatility factor for S&P index using 1-previous year historical data for each month between 1951 and 2016. We found the results in the table below:

Table 5 : Correlation between Excess Volatility and Excess Returns of S&P 500 index

Max(EV)	0,373827011
Min(EV)	7,35061E-06
Moy(EV)	0,034107409
Corr(EV,ER)	18,2805%

The Max (EV), Min (EV) and Moy (EV) are respectively the maximum, the minimum and the average of monthly excess volatility of S&P 500 index. We found positive correlation Corr (EV, ER) = 18.2805 % between monthly excess returns and monthly excess volatility of S&P 500 index. This confirms our findings. As I mentioned we can do the same for all stocks listed in US market and compute the average values. After validating the mean reversion process of stock prices towards their fundamental values which is the basis of our excess volatility factor proxy, and verifying the positive correlation between excess returns and excess volatility, we need now to regress returns on returns within the framework of validating the new model. The methodology to follow is the set of

steps we explained in the previous section, in this part of empirical results we have to explore results we get from these manipulations, but we don't have all necessary historical data we need for that.

III. WANG AND MAC MODEL WITH EXCESS VOLATILITY FACTOR

In 2014 Wang and Mac introduced new equity risk premium model with excess volatility factor, it was the first one and the only one since. I set below the main steps that Wang and Mac followed in their research paper to conceive the new equity risk premium model with excess volatility factor.

- Wang and Mac consider a proxy for excess volatility factor which has been used by Mackinlay and all in their research paper in 1988 to discuss excess volatility of stock prices. The proxy consists on approaching the excess volatility by the difference between q times 1-day volatility and q-days volatility, $VD\,(q) = q * \sigma_1^2 - \sigma_q^2$ where $\sigma_1^2 = \sum_{k=1}^{n} \frac{(P_k - P_{k-1} - \hat{\mu})^2}{n-1}$ is volatility of one day based on one previous year historical data or 12 previous month historical data as the variance difference is assigned to one month. They used volatility of daily logarithmic return,

  $\hat{\mu} = \frac{1}{n}\sum_{k=1}^{n} P_k - P_{k-1}$ is mean of daily returns in basis of 12 months historical data, $\sigma_q^2 = \sum_{k=q}^{n} \frac{(P_k - P_{k-q} - q\hat{\mu})^2}{m}$ is volatility in q-days where $m = (n - q + 1) * (1 - \frac{q}{n})$, it is equivalent to variance of q-days returns. In Wang and Mac q was 22, because they regress monthly returns in equity risk premium model; they were interested to integrate in this model a new factor representing excess volatility in monthly returns, which means they chose to model excess volatility in one month. The variance difference proxy in one month is the difference between 22 business day volatility and volatility in 22 business days.
- To make dynamic trading, Wang and Mac consider trading strategies executed in each month. In each month, they construct new portfolios in basis of the proxy of excess volatility they considered. For each month, they compute variance difference of all stocks listed in US market, they rank all stocks in deciles in ascending way according to their variance difference, they get ten portfolios where the first one has the lowest excess volatility and the last one has the highest excess volatility, they repeat the same trading strategies each month, compute variance difference and monthly excess returns of the ten portfolios. They get finally series of variance difference and monthly excess returns of the ten portfolios for the whole period considered in the study.
- They compute average of monthly excess returns and variance difference of the ten portfolios over time, they set the ten portfolios with their average variance difference and average monthly excess returns, they observe positive correlation between variance

difference and monthly returns as the monthly excess returns increase from the first portfolio to the tenth portfolio in a parallel way of variance difference. The positive correlation Wang and Mac found in their results congruent with the fact that the first portfolio with the lowest variance difference requires the lowest equity risk premium because it is the least risky and the tenth portfolio with the highest variance difference requires the highest excess returns because it is the riskiest one.

- After validating the positive Correlation between excess volatility and excess returns, Wang and Mac modelled the new equity risk premium with excess volatility factor, they tested CAPM and Fama-French-Cahart 4 factors (beta, firm size, growth and momentum factors) model on the ten portfolios they constructed in basis of variance difference, for each portfolio of the ten investment portfolios they regress monthly excess returns from one hand on CAPM and from another hand on the four factors of Fama-French-Cahart model but they found that both models do not fit the data of the ten portfolios that they built on basis of variance difference because the intercept among regression coefficients was statistically significant for the ten portfolios in both models. They concluded that CAPM and Fama-French-Cahart model don't model well the data of portfolios based on excess volatility factor. Then they moved to test the equity risk premium model with the four previous factors and the new excess volatility factor. They found high explanatory power of the model by regressing for all investment portfolios monthly excess returns on the five factors.

  $R_{i,t} - R_{f,t} = \alpha_1 * Beta_{i,t} + \alpha_2 * \ln(MV_{i,t}) + \alpha_3 * \ln(\frac{BV_{i,t}}{MV_{i,t}}) + \alpha_4 * MOM_{i,t} + \alpha_5 * VD_{i,t}$, all regression coefficients are statistically significant and intercepts are not statistically significant for all investment portfolios which means that this model with excess volatility factor fit the data of the ten portfolios built in basis of excess volatility factor. Wan and Ma validated this model.

Remarks:

We can compare the model that we proposed in this study and Wang and Mac model, by regressing both models, comparing the explanatory power R-squared, the statistical significance of regression coefficients and the stability of the old factors in presence of the new excess volatility factor, we need only to make sure that we have the necessary data.

In Wang and Mac model, the proxy of excess volatility is not always positive as it must be according to Shiller findings in the first research paper that dealt with stock prices excess volatility, Shiller mentioned that market stock prices are always more volatile than fundamental values, in this

basis excess volatility must always be positive which was not the case in Wang and Mac research papers since they found positive values of excess volatility for some investment portfolios and negative values for others. Moreover Shiller set that excess volatility is excess dispersion in daily prices, hence the proxy of excess volatility must refer exactly to excess in volatility of daily prices, whereas Wang and Mac excess volatility proxy measure excess dispersion in one month.

CONCLUSION

Excess Volatility is excess dispersion in stock prices as Shiller defined it in 1981; it occupied a great place in research and asset management. Our concern in this study was to discuss the global framework Excess Volatility and all aspects that surround this new common risk factor. We started by reporting the important findings about excess volatility bounds tests which are dedicated to prove the existence of excess volatility. Then we discussed some models that researchers proposed to show the contribution of some factors in creating excess volatility in stock prices. But the most important aspect in our study has been to suggest a new equity risk premium model which includes stock prices excess volatility as common risk factor.

We firstly present the general methodology that researchers have used to conceive equity risk premium models as a set of risk premiums associated proportionally to risk factors they reflect. We discussed some arguments that make excess volatility behaving as common risk factor. Excess volatility is not observable in the market, but to integrate it in equity risk premium models it must be measurable, the most rational insight is to find a proxy for excess volatility. We went back to some properties of stock prices in the market. In market microstructure, stock prices are noticeably behaving according to mean reversion process towards their fundamental values, this property is not built only on observations but it has been proved by several researchers and in 1995 Raymond Chiang, Peter Liu, John Okunev discussed this issue in their research papers and set that stock prices are mean reverting around their fundamental values. We started from this property of stock prices which is also congruent with factors that are responsible on excess volatility and we found a proxy to stock prices excess volatility. This proxy is always positive as excess volatility definition requires and all parameters in its formula are observable in the market.

As we found a proxy for excess returns, we suggested a new equity risk premium model with the three factors of Fama and French models, market risk, firm size and firm growth and we added excess volatility factor measured by its new proxy. Before regressing equity risk premium model, we must firstly be sure that data from the market fits the mean reversion process that we used to find excess volatility proxy, after making sure of that we have to check the positive correlation between excess returns and excess volatility measured by its proxy, we did that for S&P 500 index, we found that US market index prices fits mean reversion process with positive mean reversion speed and excess returns of S&P 500 are positively correlated to excess volatility. This model we suggested was not the first one, Wang and Mac proposed in their research paper in 2014 another equity risk premium model but with different proxy associated to stock prices excess volatility, to compare these two models, we can gather data for all individual stocks in the market and regress excess

returns using both models, then we compare their explanatory powers and the significance of regression coefficients.

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The data used in empirical study is picked from Robert Shiller Website below:

< http://aida.wss.yale.edu/~shiller/data/chapt26.html> (consulté le 15 Mai 2017).

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