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Gemma Harris. Efficiency and distributional effects of rising public debt. Economics and Finance. 2022. dumas-04188006

HAL Id: dumas-04188006

<https://dumas.ccsd.cnrs.fr/dumas-04188006>

Submitted on 25 Aug 2023

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PARIS SCHOOL OF ECONOMICS
ÉCOLE D'ÉCONOMIE DE PARIS

MASTER THESIS N° 2022 – 02

Efficiency and distributional effects
of rising public debt

Gemma Harris

JEL Codes: D31, E21, H31, H60, L26.

Keywords:

L'ÉCOLE
DES HAUTES
ÉTUDES EN
SCIENCES
SOCIALES



PSL
RESEARCH UNIVERSITY PARIS

UNIVERSITÉ PARIS 1
PANTHÉON SORBONNE



Efficiency and distributional effects of rising public debt

Gemma Harris

Supervised by Axelle Ferriere



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ECOLE D'ECONOMIE DE PARIS

Master Thesis

Analysis and Policy in Economics
Paris School of Economics
June 2022

Efficiency and distributional effects of rising public debt

Gemma Harris*

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Abstract

Recent rises in government debt in the face of the COVID-19 pandemic, on top of already high debt-to-GDP ratios following the 2008 global economic crisis, puts the spotlight back on benefits and costs to economic efficiency and consequences for already high levels of wealth and income inequality. I develop a heterogeneous agent framework with incomplete markets and entrepreneurship that models differential savings behavior and heterogeneous returns to capital to analyze changes in public debt. Increasing debt improves overall efficiency, but the reallocation of capital away from entrepreneurs limits aggregate productivity. A tighter borrowing constraint on entrepreneur capital investment increases difficulty growing out of financial constraints. Higher debt raises household savings, with workers increasing their savings by relatively more than entrepreneurs, which slightly reduces wealth inequality. Workers, however, suffer a higher burden from losses in income and income inequality increases. Overall levels of welfare and consumption decrease, in which entrepreneur welfare gains that are dominated by the welfare losses of workers.

JEL classification: D31, E21, H31, H60, L26

***Acknowledgements:** I am very grateful to my advisor, Axelle Ferriere, for her theoretical and computational guidance, without whom this paper would not have been possible. Many thanks also go to Gilles Saint-Paul and Tobias Broer for their feedback and suggestions and to the participants of the macroeconomics workshop for their thoughtful questions and comments.

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1 Introduction

High public debt as a result of government policies in response to the COVID-19 pandemic has revitalized the debate concerning the benefits and costs of increasing debt. Economic stimulus packages to counteract the supply and demand shocks induced by the pandemic pushed debt-to-GDP ratios to levels previously unseen in many countries. Figure 1 illustrates the jump in public debt in the United States over the past two decades. The debt-to-GDP ratio increased from below 60% in 2000 to above 80% in 2010 in reaction to the global economic crisis and continued to rise, reaching 100% by 2015. With the COVID pandemic, public debt jumped again in 2020 to 120% of GDP and higher as the pandemic lengthened. Already an issue of concern following the 2008 crisis, further increases in debt levels lead to questions on the impact on economic efficiency and distributions of benefits and costs.

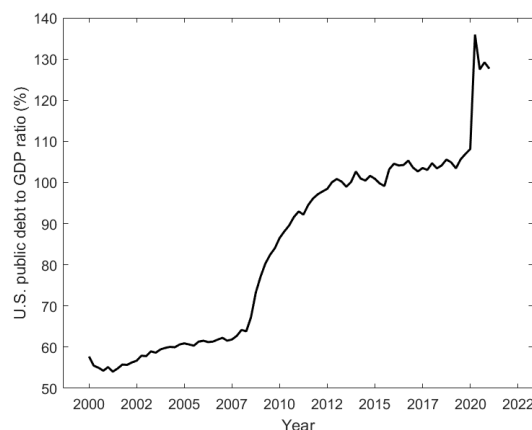


Figure 1: Total public debt as a percent of GDP, United States

In classic models of the dynamics of public debt and business cycle fluctuations, an increase in debt raises the interest rate, consequently crowding out capital. In an inefficient economy with an overaccumulation of capital, this crowding out effect, despite decreasing total production, increases efficiency by bringing capital stock closer to the optimal level. Higher debt, however, is financed by an increase in taxes, which increases distortions in the economy and contributes to a rise in inequality.

The effects of changing public debt on efficiency and distributions of wealth and income in an economy can be studied in a heterogeneous agent model with incomplete markets and precautionary savings motives, as by Aiyagari and McGrattan (1998). On one hand, increasing debt increases liquidity and improves consumption smoothing, through the mechanism of higher interest rates that incentivize higher household savings. On the other hand, the increase in the interest rate crowds out capital and decreases production, reducing welfare. Household labor incomes decline due to the decrease in wages that accompanies the increase in the interest rate. Distributional costs result from differing impacts between high and low asset households. The increase in the marginal product of capital, corresponding to the interest rate, benefits high asset households more as they earn higher capital income which offsets the decrease in labor earnings. Whether the benefits outweigh the costs, and how these effects are distributed, relies on how increasing public debt changes the accumulation of assets and returns to capital.

A growing body of literature has focused on heterogeneous capital returns and the effects of such heterogeneity on wealth accumulation and inequality. De Nardi and Fella (2017) argue that incorporating entrepreneurship in a classic Bewley-Huggett-Aiyagari heterogeneous agent

framework effectively models heterogeneous returns. Adding entrepreneurs also better captures differential savings behavior between rich and poor households, as discussed by Quadrini (2009). Taking heterogeneous returns into account, the model produces a pattern of high returns for high asset households (mostly entrepreneurs) and low returns for low asset households (mostly workers). Wealth is increasingly concentrated at the top of the distribution, as observed in the data.

I develop a heterogeneous agent model with entrepreneurs to investigate the effects of increasing government debt on the allocation of capital and the heterogeneity of returns and thus on economic efficiency and distributions of wealth. Incorporating entrepreneurs adds key features to the model:

1. Entrepreneurs operate small heterogeneous firms in a separate sector of production, which effectively endogenizes entrepreneurs' marginal products of capital and corresponding rates of return on investment.
2. Entrepreneurs earn profits by investing in capital to run their firms, resulting in an incentive to invest in more capital to expand production and increase profits.
3. An idiosyncratic shock to entrepreneur firm productivity creates an investment risk that entrepreneurs must self-insure against given the incompleteness of financial markets.
4. A borrowing constraint on capital investment that depends on wealth generates a second motive for entrepreneurs to save in order to overcome the borrowing constraint.

These features create heterogeneous returns to capital and contribute to differential savings behavior. While both entrepreneurs and workers have precautionary motives to save, entrepreneurs' additional incentive to overcome financial constraints causes entrepreneurs to accumulate more assets than workers. Endogenizing entrepreneurs' rates of return to capital results in higher returns for high asset entrepreneurs (scale dependence) and for entrepreneurs with a high productivity shock (type dependence). The borrowing constraint on capital heightens heterogeneous returns by reinforcing scale dependence. In the model, the constraint binds only for low asset entrepreneurs, restricting their investment and ability to grow their firm, resulting in lower returns. A rise in public debt reinforces these mechanisms, with divergent impacts on efficiency and distributions in the economy.

The impact of changes in government policy on economic efficiency operates through the accumulation and allocation of capital. In a basic Bewley-Huggett-Aiyagari model without entrepreneurs, the precautionary savings motive results in an overaccumulation of capital, causing the economy to be inefficient. As an increase in debt crowds out capital, this brings the capital stock closer to the optimal level and increases efficiency. In a model with entrepreneurs, however, there are conflicting pressures on the baseline accumulation of capital. The precautionary motives for workers and entrepreneurs act to increase savings and the available supply of capital. Entrepreneurs' additional incentive to save to overcome the borrowing constraint further increases total savings. Working in the other direction, higher levels of capital investment expose entrepreneurs to more risk, putting a downward pressure on capital stock. The net effect thus depends on entrepreneur demand for capital. If the precautionary savings motives and the incentive of entrepreneurs to grow their firms dominate entrepreneurs' aversion to risk, then there will be an overaccumulation of capital. This is the case in my model: a rise in public debt and consequent crowding out of capital brings the total capital stock closer to the optimal level and increases economic efficiency.

The allocation of capital between the traditional corporate sector and the noncorporate sector, in which entrepreneurs operate, and among entrepreneurs themselves also impacts economic efficiency. The corporate sector operates with a constant technology, such that productivity is normalized to one, while the average productivity in the noncorporate sector depends on the productivity shocks of entrepreneurs and the allocation of capital across firms of different productivities. In my simulated economy, more productive entrepreneurs invest in a larger share of capital than the least productive entrepreneurs, pushing the average productivity of the noncorporate sector above that of the corporate sector. However, the economy is far from its most efficient as the corporate sector holds a larger share of total capital. An increase in debt produces diverging effects on the allocations of capital. In the model, as debt increases from 80% to 120% of GDP, the corporate sector increases its share of capital, bringing down aggregate productivity, but the least productive entrepreneurs decrease their share of capital in the noncorporate sector, increasing noncorporate productivity. The two effects balance each other and aggregate productivity remains unchanged as public debt rises.

Within the noncorporate sector, the borrowing constraint exacerbates scale and type dependence in the model by limiting the investment of low asset entrepreneurs who experience high shocks and thus have a high demand for capital. This makes it harder for such entrepreneurs to grow out of financial constraints, amplifies heterogeneity in returns and leads to a misallocation of capital which reduces entrepreneur output. Higher public debt tightens the borrowing constraint by increasing the cost of capital, making it harder for more low asset entrepreneurs to invest in their optimal level of capital. At the same time, the higher interest rate enables high asset entrepreneurs to grow their firms faster since they earn higher returns on assets. As the crowding out effect on capital dominates the incentive to grow, however, higher debt also reduces entrepreneurs' demand for capital. This reduced demand, combined with the higher accumulation of assets following the increase in debt, causes fewer entrepreneurs to hit the borrowing constraint despite its increased strictness. As a result, higher debt in my model reduces the misallocation of capital among entrepreneurs, increasing efficiency in the noncorporate sector and thus improving overall economic efficiency.

Increases in public debt also impact the distribution of households in the economy by changing concentrations of wealth and income. On top of the precautionary savings motives, entrepreneurs' additional motive to save to increase capital investment causes entrepreneurs to accumulate more assets than workers. This widens the differences in savings rates between high and low asset households, generating high wealth inequality in the model. A rise in debt increases the savings of all households, but the associated higher return on assets disproportionately benefits high asset households, who earn higher total capital income. The increase in debt also results in lower labor earnings for workers, due to lower wages, and lower profits for entrepreneurs, due to lower capital investment. As workers have lower average savings and rely more on their labor income, they benefit less from higher returns and are hit harder by the decrease in labor income. Entrepreneurs see higher increases in income, as they have higher savings and rely less on firm profits, increasing top income shares. Nonetheless, when public debt increases in my model, workers increase their savings in risk-free assets by more than entrepreneurs, resulting in an overall decrease in wealth inequality.

Among entrepreneurs in particular, endogenous marginal products of capital that depend on scale and type generate heterogeneous capital returns. This heterogeneity in returns is magnified by the fact that the borrowing constraint limits the capital investment of low asset high shock entrepreneurs. As a consequence, capital returns are unevenly distributed across entrepreneurs, with high returns concentrated among the wealthiest, contributing to high baseline wealth inequality. An increase in debt in the model, however, causes low shock entrepreneurs to increase

their savings in assets by more than high shock entrepreneurs, which slightly decreases the concentration of wealth held by the top 10% of entrepreneurs. The losses in profits induced by higher debt impact low shock entrepreneurs the least, as earn very little income from profits to begin with, thus contributing to reduced income inequality between entrepreneurs.

The rest of the paper proceeds as follows. In section 2, I review the related literature. I then present the model in section 3 and describe the calibration strategy in section 4. In section 5 I detail the results for the baseline model and the effects of increasing public debt. Section 6 concludes and presents avenues for further research.

2 Literature

This paper is part of a recent growing literature on differential savings behavior and heterogeneous returns to capital in quantitative macroeconomic models. Accounting for both of these factors helps to explain the high concentrations of wealth and income observed at the top of distributions of households.

In the basic model of heterogeneous agents with precautionary savings developed by Bewley (1986), Huggett (1993) and Aiyagari (1994), there is one type of household, workers, who face idiosyncratic risks to labor earnings and save in a risk-free asset with a rate of return that is constant across all households. More recent studies, including Flavin and Yamashita (2002), Kartashova (2014) and others have revealed that rates of return in fact vary and can depend on the type of investment and associated risk. See De Nardi and Fella (2017) for a review of their findings. Idiosyncratic shocks that generate differential rates of return affect patterns of wealth accumulation and inequality. Benhabib et al. (2019) find that idiosyncratic rates of return, while not the only factor at play, contribute significantly to top wealth concentration. Scale dependence, in which returns depend on assets accumulated, and type dependence, such that returns depend on individual characteristics, both give rise to wealth inequality and at least one of these dependences is necessary to produce top wealth concentrations, as revealed by Gabaix et al. (2016). Integrating shocks that influence rates of return into a heterogeneous agent framework is a key feature that enables the model to better match high concentrations at the top of the wealth distribution. I incorporate this aspect in my model by adding entrepreneurs whose returns on capital investment depend on both accumulated assets and realized productivity shock.

A large body of literature has developed around the addition of entrepreneurship to the basic Bewley-Huggett-Aiyagari model. Papers such as those by Quadrini (2000), Gentry and Hubbard (2004), Buera (2008) and De Nardi et al. (2007) establish that entrepreneurs play a significant role in building top wealth concentrations. Piketty and coauthors (Piketty and Saez (2003), Piketty (2014), Piketty et al. (2018)) extensively document the rise in inequality in recent decades, especially at the top of the wealth and income distributions. As shown by Cagetti and De Nardi (2006), heterogeneous agent models with entrepreneurs perform much better than the basic model in replicating the high inequality found in the data.

The high concentration of top wealth in models with entrepreneurs stems from differential savings behavior and heterogeneous rates of return. Entrepreneurs tend to be richer than workers and save more, due to an additional savings motive to accumulate wealth in order to overcome financial constraints and invest in more capital to expand their firms, emphasized by Quadrini (2000) and Cagetti and De Nardi (2006). This matches empirical evidence that wealthier households save more than poor households, as discussed by Carroll (1998), and compounds top wealth inequality. Cagetti and De Nardi (2006) demonstrate that the addition of entrepreneurs also creates heterogeneous capital returns. Endogenizing entrepreneurs' marginal products of capital causes entrepreneurs who realize a high productivity shock to have a higher rate of return

on capital investment, creating type dependence in returns. The borrowing constraint ensures that entrepreneurs with higher accumulated assets also earn higher returns, reinforcing scale dependence. Both of these factors are studied by Luo and Zou (2008), who find that higher risk pushes entrepreneurs to accumulate more capital and that this as well as tighter financial constraints increase wealth inequalities in the economy.

The literature that incorporated entrepreneurship in heterogeneous agent models has focused on optimal tax regimes. Cagetti and De Nardi (2004) demonstrate that tax reforms that impact small entrepreneur firms, that are the most financially constrained, have the largest effects on the aggregate economy. Moreover, a less progressive income tax schedule, while increasing output, also generates higher wealth inequality. Investigating various types of income taxes, Kitao (2008) shows that decreasing the tax on capital income from savings increases total capital stock and production, at the cost of decreased firm investment. A lower tax on firm income increases entrepreneurial investment and production. The progressivity of the income tax regime is studied by Imrohorglu et al. (2018), who find that increasing overall progressivity is more effective than a tax on the top 1% in terms of reducing wealth inequality and increasing welfare. Bruggemann (2021) investigates top marginal tax rates in detail and demonstrates that welfare increases by more for entrepreneurs than for workers, revealing the distributional costs of this change in policy.

Beyond tax policy, the original quantitative heterogeneous agent model was also used to study changes in government debt and optimal debt levels. Aiyagari and McGrattan (1998) find that an increase in public debt in the form of risk-free bonds effectively loosens the borrowing constraint and increases the interest rate, which increase agents' savings, allowing them to smooth consumption and improve welfare. The increase in debt, however, also crowds out capital in the economy, decreasing production and wages, thus decreasing agents' income and consumption and reducing welfare. In their model, Aiyagari and McGrattan conclude that the optimal level of debt is positive and that the benefits and costs of increasing debt almost perfectly cancel. These results are conditional on transfers being below optimal, as noted by Floden (2001), who emphasizes the observation that the increase in the interest rate caused by the increase in debt corresponds to an increase in the marginal product of capital, which benefits wealthy households more and contributes to a rise in inequality resulting from the increase in government debt. Floden (2001) also discusses the effects on economic efficiency of increasing public debt. Precautionary savings lead to an overaccumulation of capital that is above the optimal efficient capital stock. As higher debt crowds out capital, this brings the total capital stock in the economy closer to its efficient level.

Do these results hold in a Bewley-Hugget-Aiyagari-type model that adds entrepreneurship to account for differential savings behavior and heterogeneous returns to capital? My paper fills this gap in the literature by attempting to answer this question. Economic efficiency depends on the accumulation of capital: the addition of entrepreneurs generates less overaccumulation of capital and could result in an underaccumulation if entrepreneurs' aversion to risky capital investment outweighs their precautionary motive to save, as discussed by Quadrini (2009). Aggregate productivity in the economy is influenced by the allocation of capital across firms with differing levels of productivity, causing efficiency in an economy with entrepreneurs to depend also on the allocation of capital between the traditional corporate sector and the entrepreneurial noncorporate sector and among entrepreneurs themselves who face shocks to productivity. Changing the level of public debt affects the accumulation and allocation of capital, thus impacting economic efficiency. The distributional effects of changes in debt also act through the accumulation of capital, as well as returns to capital. As in the basic model, an increase in debt increases savings and returns. With the addition of entrepreneurs, higher

debt influences the savings behavior of workers and entrepreneurs differently by weakening entrepreneurs' incentive to save to increase capital investment. An increase in the debt also increases the heterogeneity of returns, with diverging effects on inequality. Overall welfare in an economy with entrepreneurs therefore depends on which households gain or lose more following a rise in public debt.

There is a strand of the macroeconomic literature that investigates the relationship between debt and inequality. In a different framework that also includes entrepreneurs, Azzimonti et al. (2014) find that an increase in income inequality, which they induce by modelling higher entrepreneur income risk, leads to increased government debt. Departing from entrepreneurship, Mian et al. (2021) study differential savings behavior by introducing non-homothetic preferences, which creates heterogeneous marginal propensities to consume across households. They also conclude that increased inequality, measured as a rise in top income shares, increases the total debt in the economy. I am interested in the opposite direction of this link between inequality and debt: does an increase in debt increase inequality? To study the outcomes of increasing public debt on inequality, as well as on efficiency and welfare, I model an economy with heterogeneous households and an entrepreneurial sector, described in detail in the following section.

3 Model

3.1 Environment

Endowments

The economy is populated by a continuum of infinitely lived households of unit mass. There are two types i of households: either worker $i = w$ or entrepreneur $i = e$. Workers supply labor inelastically and face idiosyncratic shocks to labor productivity z^w . Labor productivity shocks can take values in $Z^w = \{z_1^w, z_2^w, \dots, z_{N_w}^w\}$ and evolve following a first-order Markov process with transition probabilities $\mathbb{P}^w(z'^w|z^w)$. Entrepreneurs do not supply labor, but instead invest in capital and demand labor from workers to run firms and face idiosyncratic shocks to firm productivity z^e . Entrepreneur firm productivity shocks take values in $Z^e = \{z_1^e, z_2^e, \dots, z_{N_e}^e\}$ and evolve following a first-order Markov process with transition probabilities $\mathbb{P}^e(z'^e|z^e)$. Despite being similar formally, the two shocks are independent of each other.

Preferences

Each household chooses its level of consumption to maximize its flow of utilities, with this objective described as

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(c_t^i, \ell_t^i), \quad \forall i \in \{w, e\}. \quad (1)$$

Future utility is discounted by households at rate β .

Technology

As in Quadrini (2000), Cagetti and De Nardi (2006) and Bruggemann (2021), there are two sectors of production.

Entrepreneurial production takes place in the noncorporate sector, in which small heterogeneous firms are run according to the production function

$$y = f(k, n) = z^e (k^\gamma n^{1-\gamma})^\nu. \quad (2)$$

Each period, entrepreneurs invest in capital k and employ n efficiency units of labor supplied by workers. The parameter γ indicates the capital share of production in the noncorporate sector and the span-of-control parameter ν captures the decreasing returns to scale of the production function, reflecting the idea that efficiency decreases as an entrepreneur expands its firm as modeled by Lucas Jr (1978). The inputs k and n , subject to the productivity shock z^e , determine the output y of each firm, effectively endogenizing the rate of return on capital for each entrepreneur. Entrepreneurs with higher productivity levels have a higher marginal product of capital and thus earn higher returns on capital investment. Profits are determined based on output and the prices of capital and labor. Capital depreciates at rate δ .

Large publicly owned firms operate in the perfectly competitive corporate sector, represented by the Cobb-Douglas production function

$$Y^c = F(K^c, N^c) = A^c (K^c)^\alpha (N^c)^{1-\alpha}. \quad (3)$$

The corporate sector takes inputs of capital K^c and effective labor N^c and uses a constant technology A^c to produce output Y^c . The rate of capital depreciation δ is the same as in the non-corporate sector.

Market arrangements

All households face incomplete markets and thus have precautionary motives to save to insure against risk. Workers can self-insure against idiosyncratic labor productivity shocks, that create earnings risk, by savings in one-period risk-free government bonds. Entrepreneurs can save in government bonds to self-insure against idiosyncratic firm productivity shocks, that pose risks to capital investment.

Entrepreneurs also have the opportunity to borrow to invest in capital in order to run their firms. Entrepreneur borrowing is subject to the constraint $(1+r)k \leq \lambda a^e$ such that investment depends on accumulated wealth. The strictness of the borrowing constraint is represented by the parameter λ . Workers may not borrow.

The asset market operates such that total savings by workers and entrepreneurs equals total capital demanded by the corporate and noncorporate sectors net of total government debt. The labor market operates such that total labor demanded by the corporate and noncorporate sectors equals the labor supplied inelastically by workers.

As the corporate sector is perfectly competitive, the marginal product of capital in the corporate sector determines the equilibrium interest rate r and the marginal product of labor in the corporate sector determines the equilibrium wage w .

Government

The government issues one-period debt B and raises revenue T from taxes on household income to finance wasted spending G and repay the previous period's debt, subject to the budget constraint

$$G + (1+r)B = B' + T. \quad (4)$$

Taxes are levied at a flat rate τ for all households. The government earns revenue from the taxes paid by workers on labor and capital income and from the taxes paid by entrepreneurs on firm profits and capital income.

Households

A worker's value function is given by

$$v^w(a^w, z^w) = \max_{\{c^w, a'^w\}} \{u(c^w, \ell^w) + \beta \mathbb{E}v^w(a'^w, z'^w)\} \quad (5)$$

subject to

$$c^w + a'^w = a^w + y^w(1 - \tau), \quad (6)$$

$$y^w = ra^w + wz^w\ell^w, \quad (7)$$

$$a'^w \geq 0, \quad (8)$$

$$\ell^w = 1. \quad (9)$$

Each period, workers know their level of assets accumulated a^w and the realization of their labor ability z^w . They choose consumption c^w and savings a'^w to maximize lifetime utility subject to their budget constraint (6). Workers are taxed at a flat rate τ on total income y^w , made up of capital income ra^w and labor income $wz^w\ell^w$ (7). They face a constraint such that they cannot borrow (8) and supply one unit of inelastic labor (9).

An entrepreneur's value function is given by

$$v^e(a^e, z^e) = \max_{\{c^e, a'^e, k, n\}} \{u(c^e, \ell^e) + \beta \mathbb{E}v^e(a'^e, z'^e)\} \quad (10)$$

subject to

$$c^e + a'^e = a^e + y^e(1 - \tau), \quad (11)$$

$$y^e = ra^e + z^e(k^\gamma n^{1-\gamma})^\nu - (r + \delta)k - wn, \quad (12)$$

$$a'^e \geq 0, \quad (13)$$

$$(1 + r)k \leq \lambda a^e, \quad (14)$$

$$n \geq 0. \quad (15)$$

Each period, entrepreneurs know their level of assets accumulated a^e and the realization of their firm productivity z^e . They choose consumption c^e and savings a'^e and profit maximizing inputs k and n to maximize lifetime utility subject to their budget constraint (11). Entrepreneurs are taxed at a flat rate τ on total income y^e , made up of capital income ra^e and profits $z^e(k^\alpha n^{1-\alpha})^\nu - (r + \delta)k - wn$. They face a constraint such that they cannot borrow in assets a^e (13), but can invest in capital subject to the credit constraint (14). The credit constraint imposes that an entrepreneur can invest in capital k , at the current price r , up to a fraction $\frac{1}{\lambda}$ of current assets a^e .

3.2 Equilibrium

Given government debt B , a stationary equilibrium is defined, for each type of household $i = w, e$, as the value functions $v^i(a^i, z^i)$, decision rules $\{c^i(a^i, z^i), a'^i(a^i, z^i), n(a^e, z^e), k(a^e, z^e)\}$, prices $\{r, w\}$, government policies τ and constant distributions of households over state variables $m^i(a^i, z^i)$, such that, given government debt, prices and taxes,

- household decisions $\{c^w(a^w, z^w), a'^w(a^w, z^w)\}, \{c^e(a^e, z^e), a'^e(a^e, z^e), k(a^e, z^e), n(a^e, z^e)\}$ solve the respective maximization problems described above,

- markets clear for labor and assets:

$$\int z^w(a^w, z^w) \ell^w dm^w(a^w, z^w) = \int n(a^e, z^e) dm^e(a^e, z^e) + N^c,$$

$$\int a^w(a^w, z^w) dm^w(a^w, z^w) + \int a^e(a^e, z^e) dm^e(a^e, z^e) = \int k(a^e, z^e) dm^e + K^c + B,$$

- prices $\{w, r\}$ equal to marginal products in corporate sector:

$$w = (1 - \alpha) A^c \left(\frac{K^c}{N^c} \right)^\alpha$$

$$r = \alpha A^c \left(\frac{K^c}{N^c} \right)^{\alpha-1} - \delta,$$

- taxes τ satisfy the government budget constraint,
- stationary distributions of workers and entrepreneurs are time-invariant:

$$m^i(a^i, z^i) = \int Q^i(a^i, z^i) dm^i(a^i, z^i)$$

where

$$Q^i(a^i, z^i) = \mathbb{1}_{\{a'^i(a^i, z^i)\}} \sum_{z'^i} \mathbb{P}(z'^i | z^i).$$

4 Calibration

I take as my benchmark economy the steady state in which the debt to output ratio is set at 0.80 and calibrate the model to the U.S. economy. I calibrate the baseline model to match data from 2010, the corresponding time period in the U.S. when public debt represented near 80% of GDP. Most parameter values are taken from the literature, notably Kitao (2008) and Imrohorglu et al. (2018), while remaining parameter values are chosen to match moments in the data. To describe the calibration processes of parameters, I follow the same structure as when describing the model in section 3. Table 1 below displays the values and sources of exogenously fixed parameters. Table 2 presents the values of calibrated parameters.

Table 1: Fixed parameters

Parameter	Value	Source
Preferences		
Risk aversion	μ	1.5 Attanasio et al. (1999)
Labor elasticity	φ	1.7 Frisch elasticity = 0.59
Utility weight of labor	χ	1 Normalization
Production		
Technology (corp.)	A^c	1 Normalization
Capital share (corp.)	α	0.33 Gollin (2002)
Depreciation	δ	0.06 Stokey and Rebelo (1995)
Borrowing constraint	λ	1.5 Kitao (2008)
Government		
Spending to GDP	G/Y	0.15 United States President, and Council of Economic Advisers (2014)

Table 2: Calibrated parameters

	Parameter	Value
Preferences		
Discount factor	β	0.95
Production		
Capital share (entr.)	γ	0.45
Span of control (entr.)	ν	0.88
Entrepreneur productivity		
Transition probabilities	$\mathbb{P}^e =$	$\begin{bmatrix} 0.87 & 0.13 & 0.00 \\ 0.21 & 0.68 & 0.11 \\ 0.16 & 0.17 & 0.67 \end{bmatrix}$

Endowments

Workers supply labor inelastically such that $\ell^w = 1$. Worker labor productivity can take five different values, $z^w \in \{z_1^w, z_2^w, z_3^w, z_4^w, z_5^w\}$. I take these values, as well as the transition probability matrix, from Kitao (2008). The full set of values describing labor productivity can be found in Section A of the Appendix.

Entrepreneur firm productivity can take three values, $z^e \in \{z_1^e, z_2^e, z_3^e\}$. In the literature on heterogeneous agent models with entrepreneurs, most papers incorporate an occupational choice in which households choose each period whether to be a worker or an entrepreneur. In this setup, the lowest entrepreneurial ability is set at $z^e = 0$ to guarantee that a household experiencing this shock will always choose to be a worker. As I abstract from an occupational choice in my model, I do not include $z^e = 0$ as a possible realization of the shock for the entrepreneur. I take the values of entrepreneur firm productivity from Kitao (2008), ignoring the first value of $z^e = 0$. The three levels correspond with the size of firms run by entrepreneurs. Entrepreneurs with a high level of productivity have a higher return on capital and so have a stronger incentive to make higher capital investments to expand their firms and earn higher profits. Regarding the distribution of capital, Quadrini (2000) finds a highly skewed distribution of entrepreneur wealth in the data. To approximate this concentrated distribution, he assigns a smaller proportion of entrepreneurs to larger firms, specifically 60% of entrepreneurs run small firms, 30% run medium-size firms and 10% run large firms. Following Luo and Zou (2008), I calibrate the transition probability matrix of the shock to entrepreneur productivity such that the invariant distribution matches the proportion of entrepreneurs that run small, medium and large firms as found by Quadrini (2000) and to match the entrepreneurs' share of total income, 0.17. Specifically, the transition probability matrix for entrepreneurs is

$$\mathbb{P}^e = \begin{bmatrix} p_{11}^e & (1 - p_{11}^e - p_{13}^e) & p_{13}^e \\ p_{21}^e & p_{22}^e & (1 - p_{21}^e - p_{22}^e) \\ p_{31}^e & (1 - p_{31}^e - p_{33}^e) & p_{33}^e \end{bmatrix}.$$

To simplify the calibration, I set p_{13}^e and p_{31}^e equal to the values in the transition matrix used by Luo and Zou (2008) and then calibrate the remaining four parameters to match the four targets of the invariant distribution and the share of income earned by entrepreneurs. The full set of values describing entrepreneur firm productivity can be found in Section A of the Appendix.

Preferences

I set the functional form of household utility to exhibit CRRA preferences and to be additively separable in consumption and labor. Households thus derive utility as

$$u(c, \ell) = \frac{c^{1-\mu}}{1-\mu} - \chi \frac{\ell^{1+\varphi}}{1+\varphi}. \quad (16)$$

The parameter μ represents the coefficient of relative risk aversion and is set to $\mu = 1.5$, as commonly used in the literature, following Attanasio et al. (1999). The inverse of the Frisch elasticity of labor, φ , is set at $\varphi = 1.7$ such that Frisch elasticity equals 0.59, standard in this body of macroeconomics literature. However, as I simplify my model so that workers supply labor inelastically, this value does not actually affect the results of the simulations. The disutility of labor is normalized to $\chi = 1$. Again, as the supply of labor is inelastic, this parameter does not change the outcome of the model.

The discount factor β is calibrated to match the capital to output ratio of 2.9, as in Imrohorglu et al. (2018).

Technology

In the noncorporate sector, the entrepreneurial production function is

$$f(k, n) = z^e (k^\gamma n^{1-\gamma})^\nu.$$

The income share of capital γ and the span-of-control parameter ν are calibrated to match wealth and income related targets. The moments specifically targeted here are the ratio of median net worth between entrepreneurs and workers, between 5.3–6.5, following Imrohorglu et al. (2018), and the share of entrepreneur income earned by the top 10% of entrepreneurs, 0.64, as in Bruggemann (2021).

In the corporate sector, large perfectly competitive firms operate according to the production function

$$F(K^c, N^c) = A^c (K^c)^\alpha (N^c)^{1-\alpha}.$$

Technology is normalized to $A^c = 1$ and the capital share of production is set at $\alpha = 0.33$, as found by Gollin (2002) and commonly used in the macroeconomics literature.

In both sectors, the depreciation rate is set at $\delta = 0.06$, as determined by Stokey and Rebelo (1995).

Market arrangements

The borrowing constraint on capital investment by entrepreneurs is

$$(1+r)k \leq \lambda a^e.$$

Following Kitao (2008) and Bruggemann (2021), entrepreneurs can borrow up to 50% of their assets, implying a value of $\lambda = 1.5$ such that an entrepreneur's investment in capital to run its firm can be up to 1.5 times its accumulated savings.

Government

The ratio of wasted government spending to GDP is set at $G/Y = 0.15$, as in Bruggemann (2021), based on the report published by the United States President, and Council of Economic Advisers in 2014. When comparing steady state equilibria to study to effect of changing public debt, debt B is calibrated such that the debt-to-GDP ratio is set to the desired level, e.g. 0.80 in the benchmark economy.

5 Benchmark economy

5.1 Empirical comparison

I first examine baseline results in the calibrated economy, with public debt set at 80% of GDP. It is important that key moments of the data are replicated by the model in order to present a relevant analysis. Namely, the benchmark economy should match the empirical observations that entrepreneurs are wealthier than workers, entrepreneurs save more and they run heterogeneous firms. Data on targeted moments and the corresponding results generated by the calibrated model are shown in Table 3. Both in the data and in the model, entrepreneurs have a much higher median net worth than workers and receive more than 10% of total income despite making up less than 10% of the total population. Entrepreneurs themselves are not evenly distributed across sizes of firms: 60% run small firms, 30% run medium-size firms and 10% run large firms. Regarding the resulting wealth distribution among entrepreneurs, the top 10% hold more than half of total entrepreneurial wealth. This implies a high level of inequality among entrepreneurs, which compounds top wealth inequality across all households.

Table 3: Empirical targets

	Target	Model
<hr/> <hr/> Overall economy		
Capital-output ratio	2.9	2.90
Ratio of median net worth E/W	5.3-6.5	5.90
Entrepreneurs' share of total income	0.17	0.224
<hr/> Entrepreneurs		
Top 10% wealth share	0.64	0.654
Share of entrepreneurs		
with small firms	0.60	0.603
with medium firms	0.30	0.298
with large firms	0.10	0.099

I also compare the wealth and income distributions generated by the model with empirical distributions to assess the performance of the model against moments that were not specifically targeted. Table 4 shows that the results of the calibrated model produce a high level of income and wealth inequality, even more than is observed in the data. This large degree of inequality is the consequence of adding entrepreneurs as a second type of heterogeneous agent to the basic Bewley-Hugget-Aiyagari model. Entrepreneurs accumulate more assets than workers, as they have the additional motive to save to overcome the borrowing constraint on capital, leading

to stark differences in wealth between workers and entrepreneurs. Despite higher asset accumulation in general, some entrepreneurs remain unable to grow out of the imposed financial constraints. Entrepreneurs also face different marginal returns to capital, influencing their incentives to invest in capital and the extent to which they grow their firms. Both of these factors create high inequality among entrepreneurs and amplify top wealth and income inequality in the whole population.

Table 4: Wealth and income distributions

	Wealth		Income	
	Top 1%	Top 10%	Top 1%	Top 10%
<hr/>				
All households				
Data	34.1	74.4	17.2	44.4
Model	64.0	96.1	61.6	83.3
<hr/>				
Entrepreneurs				
Data	25.5	64.1	21.1	55.1
Model	44.7	65.4*	20.8	62.5

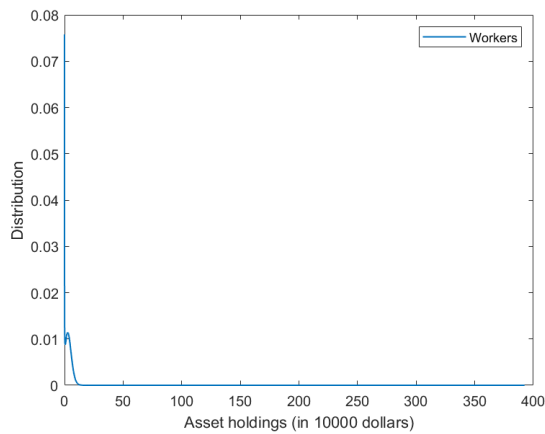
* Targeted.

5.2 Asset accumulation

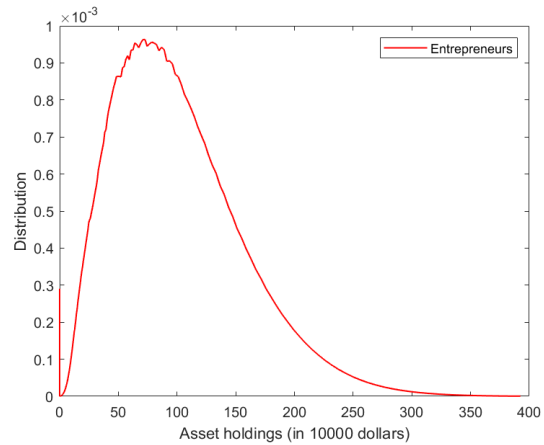
Looking at saving behavior, Figure 2 shows that entrepreneurs accumulate much more assets in savings than workers. Entrepreneurs also hold 63% of total wealth in the model. This confirms that the wealthier households save more, resulting in high top wealth inequality. Moreover, high-productivity entrepreneurs save more on average than low-productivity entrepreneurs, as demonstrated in panel (c), given their strong incentives to grow their firms.

In addition to saving in risk-free assets, entrepreneurs also invest in capital subject to the borrowing constraint that causes capital holdings to depend on savings. The accumulation of capital affects the economic efficiency of the noncorporate sector. The borrowing constraint restricts the capital investment of low asset and high shock entrepreneurs. Such households have a strong incentive to invest given their higher marginal product of capital, due to a high realization of firm productivity, but do not possess enough savings to permit them to invest in the level of capital that they demand. However, as entrepreneurs tend to have high savings, and high shock entrepreneurs save the most, there are relatively few entrepreneurs who actually hit the borrowing constraint in the benchmark steady state equilibrium.

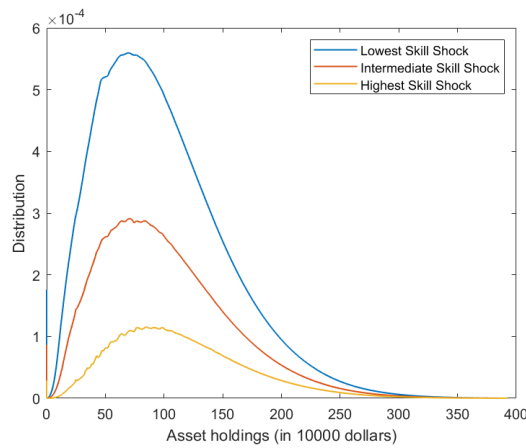
In regards to the entrepreneurs who are constrained, Figure 3 presents the percentages of entrepreneurs that hit the borrowing constraint and are hence unable to invest in their optimal level of capital. Panel (a) displays the breakdown by savings level, with low savings defined as the range of assets held by the bottom third of entrepreneurs (0-38), intermediate savings being the range held by the middle third (39-86) and high savings the range held by the top third of entrepreneurs (87-400). As shown, only low asset entrepreneurs hit the borrowing constraint, demonstrating how the constraint reinforces scale dependence in the accumulation of capital.



(a) Worker asset distribution



(b) Entrepreneur asset distribution



(c) Entrepreneur asset distribution, by type

Figure 2: Household savings distributions

To investigate type dependence, panel (b) of Figure 3 displays the breakdown by shock level. Despite high shock entrepreneurs being more likely to demand more capital and hit the borrowing constraint, they are also more likely to accumulate more savings as they make higher profits and have more income to save in risk-free assets. As a result, there are very few entrepreneurs who experience a high shock and have low accumulated assets in equilibrium. As panel (b) shows, more entrepreneurs with a intermediate level productivity shock actually hit the borrowing constraint as they do not have enough assets to overcome the constraint on capital investment. Overall, low asset entrepreneurs who experience intermediate or high productivity shocks have difficulty growing out of financial constraints: they cannot invest in their demanded level of capital in order to earn enough profits to increase savings, which would allow them to invest in more capital and continue to grow their firms. This reduces the efficiency of the noncorporate sector as all firms are not operating at their optimal level. In particular, the entrepreneurs who tend to be limited are those who would run more productive firms given the opportunity.

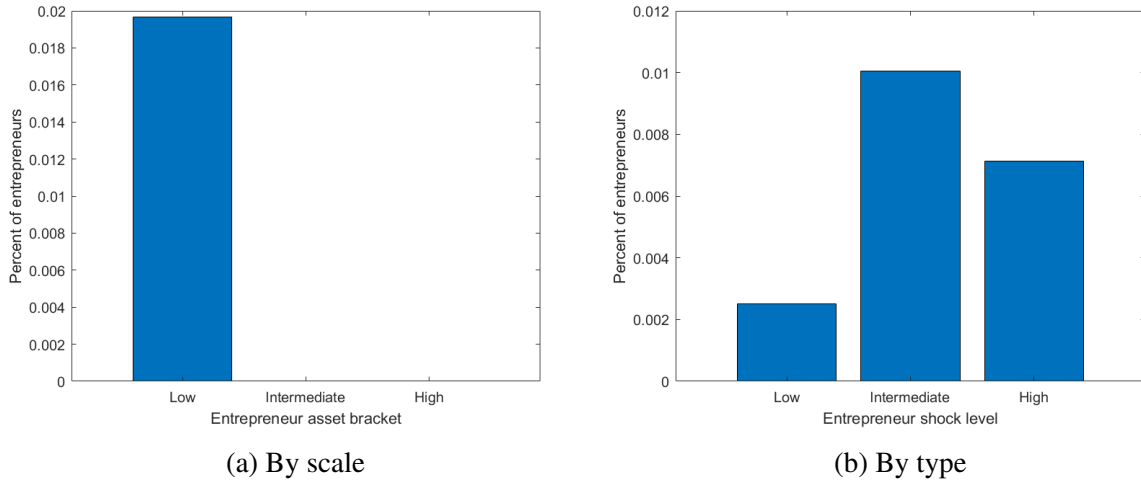


Figure 3: Percent of borrowing constrained entrepreneurs

5.3 Capital allocation

As seen above, financial constraints limit the accumulation of capital by entrepreneurs and thus reduce the efficiency of the noncorporate sector. The constraints faced by entrepreneurs also impact the allocation of capital between the corporate and noncorporate sectors, with consequences for the overall level of efficiency in the economy. Moreover, breaking down how capital is allocated across entrepreneurs themselves reveals further consequences for economic efficiency, as well as for the distribution of wealth and income across entrepreneurs.

Different realizations of productivity shocks create heterogeneous returns to capital for entrepreneurs. The most efficient allocation of capital would be to give all of the capital supplied to the most productive firms. These are the firms run by entrepreneurs receiving the highest shock and who thus have the highest marginal product of capital. Figure 4 below shows the allocation of capital between the corporate and noncorporate sectors and within the noncorporate sector. Clearly, capital is not allocated optimally across firms and the economy is not operating at its most efficient. Of total capital, 88.5% is used in production in the corporate sector compared to only 11.5% in the noncorporate sector, as shown in panel (a). This is partly because the noncorporate sector is small, as entrepreneurs make up only 7% of the population, and some entrepreneurs are borrowing constrained and thus cannot invest in as much capital as they demand. Furthermore, among entrepreneurs, capital is not allocated only to the most productive firms. Panel (b) shows that the most capital is used by firms that have an intermediate level of productivity, who invest in 58.3% of total entrepreneurial capital, whereas highly productive firms invest in 38.5% and low-productivity firms use 3.2%. Although the most productive entrepreneurs demand the most capital, they make up only 10% of all entrepreneurs, which implies that there are not enough large entrepreneur firms to invest in more capital. In addition, entrepreneurs with intermediate and high levels of productivity are also more likely to be restricted in their capital investment by the borrowing constraint.

The allocation of capital across entrepreneurs also affects entrepreneurs income and wealth distributions. Figure 5 presents the capital holdings and profits of entrepreneurs by level of savings, broken down by shock. Panel (a) illustrates that low shock entrepreneurs demand a low level of capital, whereas high shock entrepreneurs demand the highest level of capital, with optimal capital demands shown by the horizontal sections of the curves. This is a direct consequence of the heterogeneous marginal products of capital, and thus heterogeneous rates of

return, across entrepreneurs. In addition, panel (a) shows that at very low levels of accumulated assets, entrepreneurs are unable to invest in their optimal levels of capital due to the borrowing constraint, shown by the sections of the curves that are below the optimal level and increasing in assets. High shock entrepreneurs are the most bound by financial constraints, as they demand the most capital and thus need to accumulate more savings to overcome the constraint. Panel (b) reveals the effects of differential capital accumulation in terms of profits. Clearly, high shock entrepreneurs earn the highest profits, despite facing more binding constraints. High shock and high asset entrepreneurs, therefore, accumulate the most capital, allowing them to run the largest firms, earn the highest profits and continue to increase their wealth, resulting in high top wealth inequality.

Of note, Figure 5 shows capital and profits only for the lowest asset bracket (assets held by the bottom third of entrepreneurs), as these are the only households impacted by the borrowing constraint as discussed above.

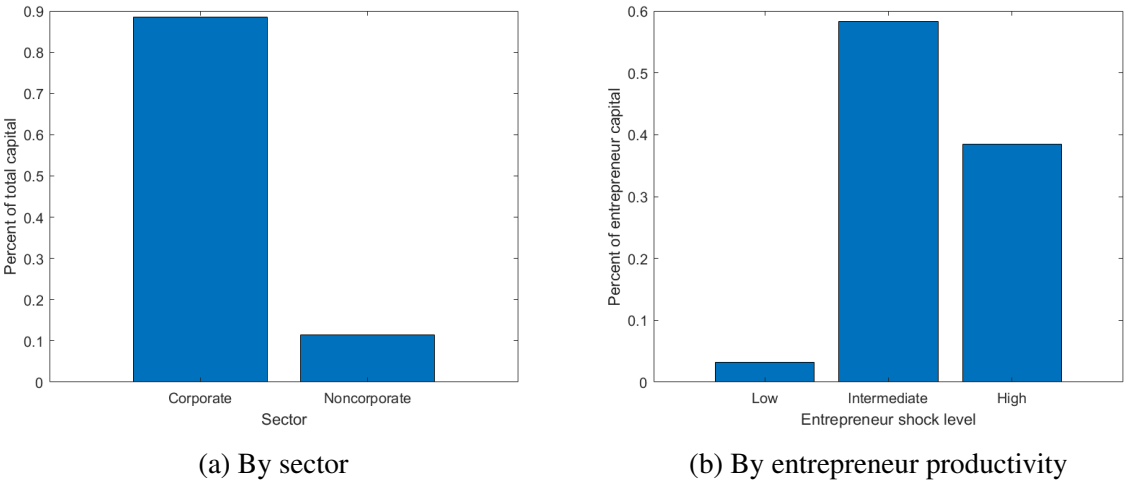


Figure 4: Capital allocation

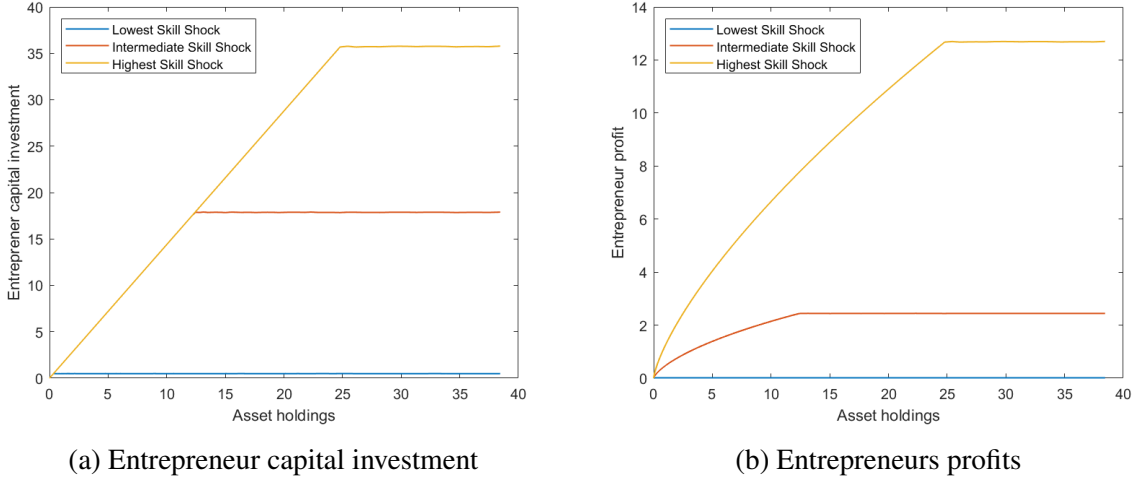


Figure 5: Capital allocation and profits across entrepreneurs

6 Increasing public debt

I next compare steady state equilibria for increasing levels of the public debt. I study the steady state results for debt-to-GDP ratios of 80%, the benchmark model, 100% and 120%. These increases match the increases in the debt-to-GDP ratio in the U.S. from 2010 to 2015, in the aftermath of the global economic crisis, and from 2015 to 2020, with a spike in debt in 2020 in response to the COVID pandemic. By impacting asset accumulation and allocation, increasing the debt affects economic efficiency and wealth and income disparities in the economy, with diverging effects on household welfare.

6.1 Efficiency

On the whole, the total economy in the model is more efficient in steady states with higher levels of debt. Capital stock decreases across the board and both the corporate and noncorporate sectors are also more efficient, despite reallocations of capital.

The increase in public debt raises the equilibrium interest rate in the economy. This creates a crowding out effect on capital accumulation in both sectors by increasing the cost of capital. The higher interest rate also corresponds to a higher marginal product of capital, however, incentivizing entrepreneurs to demand more capital to expand production. The net effect on entrepreneur capital holdings, and thus on total capital demand in the economy, depends on whether the crowding out effect dominates the stronger investment incentive. Table 5 presents the levels of capital and the efficiency ratios for the total economy and both sectors of production separately for each level of debt. Efficiency is measured, for each sector, as the fraction of output to inputs. For the whole economy, for instance, this would be $\frac{Y}{K+N}$.

As shown in the table, total capital stock decreases, driven by decreases in corporate capital and decreases in entrepreneur capital demand. Conversely, efficiency ratios increase in the overall economy and in both the corporate and noncorporate sectors. This could be because there is an overaccumulation of capital in the benchmark equilibrium, the result of high savings by workers and entrepreneurs due to their precautionary motives and the incentive of entrepreneurs to accumulate assets to overcome the borrowing constraint and invest in more capital. Crowding out capital would thus bring the capital stock closer to its optimal level and increases efficiency. However, other factors are also at play, including the effect of the borrowing constraint on the accumulation of capital by entrepreneurs and the allocation of capital between the corporate and noncorporate sector as well as between entrepreneur firms within the noncorporate sector.

Table 5: Capital accumulation following increases in public debt

	Debt to GDP ratio		
	80%	100%	120%
Total economy			
Capital	5.48	5.39	5.30
Efficiency	0.292	0.294	0.297
Corporate sector			
Capital	4.85	4.77	4.60
Efficiency	0.263	0.266	0.268
Noncorporate sector			
Capital	0.631	0.620	0.609
Efficiency	0.496	0.499	0.502

The increase in public debt affects capital accumulation in the noncorporate sector specifically by altering financial constraints. The increase in the interest rate tightens the borrowing constraint, implying that low asset entrepreneurs face more difficulty growing out of constraints. At the same time, the higher interest rate increases the returns on assets, enabling high asset entrepreneurs to grow faster. Given higher returns, entrepreneurs increase their savings in response to an increase in debt, resulting in fewer entrepreneurs with low assets. Taking into account as well the lower demand for capital due to the crowding out effect, despite the stricter constraint, fewer entrepreneurs overall are limited in their investment of capital. Figure 6 displays the percent of entrepreneurs who are borrowing constrained in each steady state equilibrium. The trends by asset level (scale) and shock level (type) presented in Figure 3 are the same for each level of debt, relative to total constrained entrepreneurs. Since a smaller percentage of entrepreneurs are borrowing constrained for higher debt-to-GDP ratios, as can be seen in the graph, increasing the debt acts to improve efficiency in the noncorporate sector as less entrepreneurs have difficulty growing out of financial constraints.

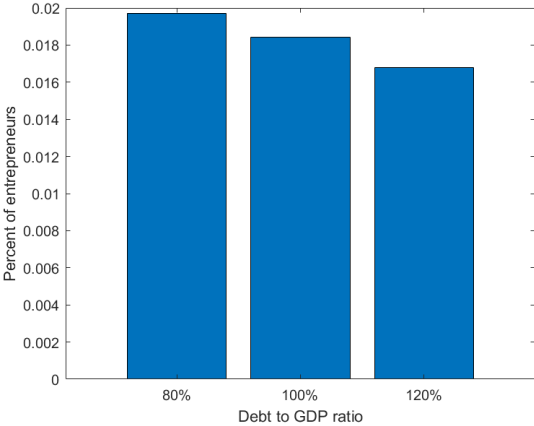


Figure 6: Percent of borrowing constrained entrepreneurs

Within the overall decreases in capital stock, higher debt changes the allocation of capital, further impacting economic efficiency. Figure 7 presents how capital allocation between the corporate and noncorporate sectors and between entrepreneurs firms changes with each increase in the debt-to-GDP ratio in comparison to the benchmark equilibrium. In terms of allocation between sectors, panel (a) illustrates that, relative to the total amount of capital in the economy, the share of capital used in production in the corporate sector increases while the share used by the noncorporate sector decreases. As the noncorporate sector has, on average, a higher productivity than the corporate sector (see Table 6) due to more capital allocated to intermediate- and high-productivity firms, this puts a downward pressure on aggregate productivity and overall efficiency. Panel (b), on the other hand, reveals that, relative to the amount of capital used in the noncorporate sector, highly productive firms increase their share of capital and less productive firms use less capital, increasing average productivity in the noncorporate sector.

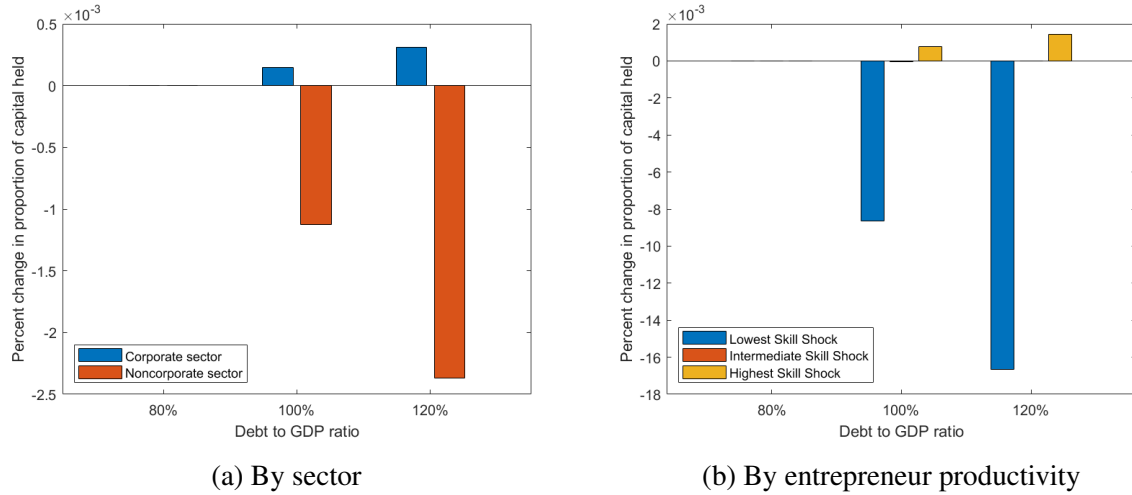


Figure 7: Change in capital allocation

Table 6 displays the levels of productivity for each sector and for the total economy in each steady state equilibrium. As public debt increases, the reallocation of capital away from the noncorporate sector is made up for by reallocation within the noncorporate sector to more productive firms. The two opposing effects of the changing allocations of capital cancel, such that aggregate productivity in the total economy remains the same. Overall efficiency nonetheless increases in both sectors and in the total economy, as shown above in Table 5. Since there are no efficiency gains from changes in aggregate productivity, this suggests that the increases in efficiency following increase in public debt in the model are fully due to the crowding out effect on capital by bringing capital stock closer to its optimal efficient level.

Table 6: Productivity following increases in public debt

	Debt to GDP ratio		
	80%	100%	120%
Total economy			
Aggregate productivity	1.085	1.085	1.085
Corporate			
Productivity	1.000	1.000	1.000
Noncorporate			
Average productivity	1.738	1.739	1.740

6.2 Distributions

Higher levels of public debt impact the distributions of wealth and income across workers and entrepreneurs through changes to asset accumulation and income earned. As taxes are imposed on total income at a flat rate across all households, and are primarily used to balance the government's budget constraint for a given level of debt, I leave an analysis of the redistributive effects of taxes for future research. Table 7 presents results on income and wealth shares. All households save more assets, with workers increasing savings by more than entrepreneurs, contributing to slightly reduced entrepreneur and top wealth concentrations. Regarding income, workers earn less from labor and entrepreneurs make lower profits. However, the decreases in labor earnings hurt workers more than the decreases in profits do for entrepreneurs, as workers rely strongly on labor income while entrepreneurs earn more from capital income. These dynamics result in a slight increases in entrepreneur and top income shares as debt rises.

Table 7: Wealth and income distributions: increasing debt

	Wealth		Income	
	Top 10% share	Entrepreneur share	Top 10% share	Entrepreneur share
All households				
B/Y=80%	96.1	63.5	83.2	22.4
B/Y=100%	95.8	63.1	83.4	22.9
B/Y=120%	95.6	62.6	83.5	23.3
Entrepreneurs				
B/Y=80%	65.4		62.5	
B/Y=100%	64.6		61.6	
B/Y=120%	63.7		60.7	

As the increase in debt increases the interest rate, all households in the model increase their savings in risk-free assets. However, increases are not equal across all households. Figure 8 shows the change in savings broken down by type of household. Workers increase their savings by more than entrepreneurs, as shown in panel (a). Although entrepreneurs still save more than workers, their share of total risk-free assets in each steady state decreases with higher levels of public debt. This is due to a weaker incentive to save to invest in capital, since entrepreneurs demand less capital due to the higher cost. Combined with the increase in the interest rate, workers' larger increase in savings causes them to gain relatively more from increases in capital income induced by increases in debt. Table 8, below, shows that workers' capital income ra^w increases by 20.5% when comparing steady states with debt-to-GDP at 80% and at 120%, whereas total capital income ra^e earned by entrepreneurs increases by only 11.2%.

Households also earn income from contributing to production. A higher interest rate is accompanied by a lower wage, reducing labor income for workers. Despite higher marginal products of capital, entrepreneurs invest in less capital and so earn less profits from running their firms. Losses are worse for entrepreneurs than for workers, with entrepreneur profits decreasing by 2.17% and worker labor income decreasing by 1.11% as debt increases from 80% to 120% of GDP, as shown in Table 8. However, as entrepreneurs hold more assets than workers, the loss in profits has a weaker effect on change in total pre-tax income for entrepreneurs than the loss in labor earnings does for workers. In other words, the gains workers receive in capital income barely offset their losses due to a lower wage. As a result, workers' total pre-tax income increases by only 0.45% while entrepreneur income increases by 5.07%. This increases income inequality between entrepreneurs and workers: Table 7 presents the share of total income held by entrepreneurs, which increases from 22.4% in the benchmark equilibrium to 23.3% in the steady state with a debt-to-GDP ratio of 120%.

On the other hand, since workers increase their savings in risk-free assets by more than entrepreneurs, this contributes to relatively larger increases in wealth for workers. Table 8 shows that workers' wealth increases by 9.22% as the debt-to-GDP increases from 80% to 120%, while entrepreneurs' wealth increases by 5.19%. Therefore, wealth inequality decreases slightly with higher levels of debt in the model economy and entrepreneurs' share of total wealth decreases by 0.90%, as shown in Table 7.

The distributions of wealth and income across entrepreneurs themselves are also impacted by the increase in public debt. Higher levels of debt affect savings behavior of entrepreneurs differently and cause capital to be reallocated among entrepreneurs, as discussed in Section 6.1, which has divergent outcomes due to heterogeneous returns to capital.

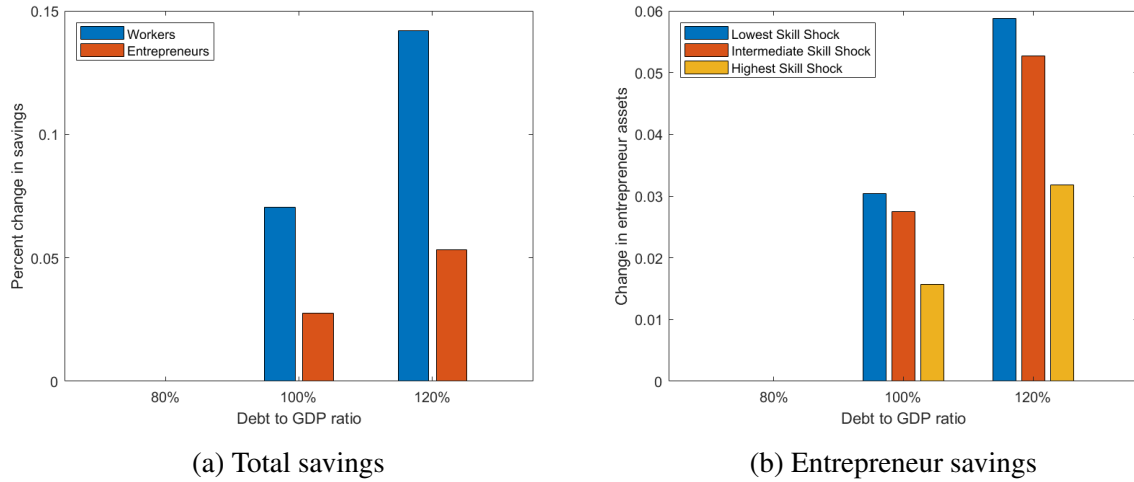


Figure 8: Changes in savings in risk-free assets

Regarding entrepreneur savings in risk-free assets, as shown in panel (b) of Figure 8, low shock entrepreneurs increase their savings by more than high shock entrepreneurs. This is the combined result of less productive entrepreneurs having a stronger precautionary motive to save, as they experience a low shock that reduces potential to earn high profits, and the increase in debt increasing the level of available risk-free assets which give higher returns. Consequently, less productive entrepreneurs increase their capital income earned on savings by more than highly productive entrepreneurs, although the most productive still have the highest average savings of all types of entrepreneurs as they earn the most income and thus the highest capacity to save. Table 9 presents this result in detail: when the debt-to-GDP ratio increase from 80% to 120%, the capital income of low shock entrepreneurs increases by 11.73%, by 11.09% for entrepreneurs who realize an intermediate shock and for high shock entrepreneurs by 8.89%. This reduces income inequality among entrepreneurs.

On the other hand, as debt increases, capital in the noncorporate sector is reallocated from less productive to more productive firms. Total capital used in production by entrepreneurs decreases but, as demonstrated in panel (b) of Figure 7, high shock entrepreneurs invest in a higher share of this capital. Layering on top of this higher marginal returns to capital due to high productivity, high shock entrepreneurs see the smallest decreases in profit. This is confirmed in Table 9: firm income from profits decreases by 1.89% for entrepreneurs with high productivity, by 2.40% for intermediate productivity entrepreneurs and decreases the most for least productive entrepreneurs by 4.35% when comparing the steady state with debt at 120% of GDP to the benchmark equilibrium. These differential changes in profit counteract the reduction in income inequality stemming from changes in capital income received on savings. However, as less productive firms operate at very low levels, the entrepreneurs who run these firms earn most of their income from returns on savings and not from profits. Their losses in profits have a minimal effect on changes in their total pre-tax income and therefore do not dominate the large increase in capital income. Conversely, highly productive entrepreneurs lose much more due to decreases in profits, with their gains from capital income almost outweighed by losses in profits. Overall, the least productive entrepreneurs see the highest relative gains in income, an increase of 11.48% when debt increases to 120% of GDP, while the most productive entrepreneurs have the lowest increase in income of 0.41%, serving to decrease overall income inequality between entrepreneurs in the model. Table 7 confirms that the share of income held by the top 10% of entrepreneurs decreases from 62.5% to 60.7%

Higher increases in income, combined with higher increases in savings, cause low shock entrepreneurs to experience the largest increases in wealth. When the debt-to-GDP ratio increases from 80% to 120%, the wealth of the least productive entrepreneurs increases by 5.98% compared to an increase of 2.53% for the most productive entrepreneurs. This reduces wealth inequality among entrepreneurs alongside income inequality, with the share of wealth held by the top 10% of entrepreneurs decreasing from 65.4% to 63.7%, as shown in Table 7.

Table 8: Wealth and income (in 1000 dollars)

	Debt to GDP ratio			Percent change from 80% to 120%
	80%	100%	120%	
All households (mass=1)				
Total wealth	82.379	85.157	87.865	6.66%
Total pre-tax income	15.799	15.921	16.048	1.58%
Capital income	2.939	3.143	3.348	13.9%
Labor/firm income	12.860	12.779	12.700	-1.24%
Workers (mass=0.93)				
Total wealth	29.279	31.352	32.743	9.22%
Total pre-tax income	11.971	11.995	12.025	0.45%
Capital income	0.864	0.951	1.041	20.5%
Labor income	11.107	11.044	10.984	-1.11%
Entrepreneurs (mass=0.07)				
Total wealth	52.400	53.805	55.122	5.19%
Total pre-tax income	3.829	3.926	4.023	5.07%
Capital income	2.075	2.192	2.307	11.2%
Firm income	1.754	1.735	1.716	-2.17%

Table 9: Entrepreneur wealth and income (in 1000 dollars)

	Debt to GDP ratio			Percent change from 80% to 120%
	80%	100%	120%	
Low productivity entrepreneurs (60%)				
Total wealth	28.932	29.826	30.663	5.98%
Total pre-tax income	1.195	1.264	1.332	11.48%
Capital income	1.176	1.246	1.314	11.73%
Firm income	0.019	0.018	0.018	-4.35%
Intermediate productivity entrepreneurs (30%)				
Total wealth	15.936	16.354	16.737	5.03%
Total pre-tax income	1.349	1.375	1.401	3.85%
Capital income	0.625	0.660	0.694	11.09%
Firm income	0.724	0.715	0.707	-2.40%
High productivity entrepreneurs (10%)				
Total wealth	7.532	7.626	7.723	2.53%
Total pre-tax income	1.285	1.287	1.290	0.41%
Capital income	0.274	0.286	0.298	8.89%
Firm income	1.011	1.001	0.992	-1.89%

6.3 Welfare

Turning to outcomes for welfare and consumption in the model economy, steady state equilibria with higher levels of debt result in slightly lower total consumption, with higher variation in consumption levels across households, and decreased overall welfare. Welfare is studied using the utilitarian welfare criterion:

$$W^i = \int_{a^i, z^i} v^i(a^i, z^i) dm^i(a^i, z^i), \forall i \in \{w, e\},$$

where $v^i(a^i, z^i)$ is a household's value function and $m^i(a^i, z^i)$ is the time-invariant equilibrium distribution of households over assets and productivity levels. Total welfare in the economy is thus $W^{tot} = W^w + W^e$. Consumption equivalence, reflecting changes in welfare across steady states, is calculated for each type of household as the amount by which consumption in the benchmark equilibrium would need to be permanently changed such that households are indifferent between the benchmark steady state and the new steady state with a higher debt-to-GDP ratio:

$$CEV^i(a^i, z^i) = \int_{a^i, z^i} \left[\left(\frac{v_{new\ SS}^i(a^i, z^i)}{v_{benchmark\ SS}^i(a^i, z^i)} \right)^{\frac{1}{1-\mu}} - 1 \right] dm^i(a^i, z^i), \forall i \in \{w, e\}.$$

See Section C of the Appendix for details on the derivation of consumption equivalence. Changes in public debt also affect the sensitivity of changes in consumption to changes in income, as steady state income rises with increases in debt in the model. Consumption variation gives an indication of the ability of households to smooth consumption. It is calculated as the variance of log-consumption across households, as consumption is a nonlinear increasing function of savings in assets with high curvature at low asset levels and close to linear for high asset levels.

Table 10 presents results on consumption variation, adjustments in consumption across steady states, consumption equivalence for each type of household between steady states and changes in welfare. For the entire economy, looking at all households together, total welfare decreases by 0.30% with each increase in the debt-to-GDP ratio, from 80% to 100% and from 100% to 120%. This is due to decreases in household consumption, by 0.71% and then by 0.51%, and increased variance in log-consumption, by 0.35% and 0.69%, as debt increases from the benchmark to 120% of GDP.

Breaking down these aggregate effects into outcomes for workers and entrepreneurs separately reveals that entrepreneurs see increases in consumption and welfare while workers experience decreases. Variation in worker consumption significantly decreases, which contributes a positive effect to worker welfare as households prefer to smooth consumption, but total worker consumption decreases by just over 1% with each increase in public debt. As shown in Table 8, the total income of workers increases only slightly as gains in capital income barely compensate for losses in labor earnings. This suggests that worker consumption is more sensitive to labor income: as labor income decreases, workers save more as a form of self-insurance (see Figure 8, panel (a)) and consume less. Welfare of workers decreases by relatively more than the decline in total welfare in the economy, a drop that indicates that workers are better off in the benchmark compared to the other steady states. This is reflected in negative measures of consumption equivalence, which show that worker consumption would have to be permanently reduced in the benchmark equilibrium for workers to be indifferent between the benchmark and steady states with higher levels of debt. Conversely, permanent increases in worker consumption would be necessary in steady states with larger debt-to-GDP ratios to make workers indifferent between the benchmark and jumping to such new steady states.

Table 10: Changes in welfare

	Debt to GDP ratio		
	80%	100%	120%
<hr/>			
All households (mass=1)			
Consumption variation	0	0.35%	0.69%
Change in consumption	0	-0.71%	-0.51%
Change in welfare	0	-0.30%	-0.30%
<hr/>			
Workers (mass=0.93)			
Consumption variation	0	-24.91%	-23.22%
Change in consumption	0	-1.14%	-1.01%
Consumption equivalence	0	-0.0108	-0.0217
Change in welfare	0	-0.36%	-0.36%
<hr/>			
Entrepreneurs (mass=0.07)			
Consumption variation	0	4.34%	3.04%
Change in consumption	0	0.88%	1.33%
Consumption equivalence	0	0.0002	0.0003
Change in welfare	0	0.62%	0.55%

Entrepreneurs, on the other hand, experience increases in consumption variation, but also see increases in total consumption, by 0.88% and then by 1.33%. In terms of the sensitivity of consumption to income, total entrepreneur income rises by 5.07% when public debt increases from 80% to 100% of GDP, as demonstrated in Table 8, in which gains in capital income outweigh losses in firm profits. When the government increases debt, entrepreneurs increase savings in risk-free assets and earn higher returns on these savings, invest in less capital, and are thus exposed to less risk despite losses in profits, and they consume more, implying that changes in entrepreneur consumption are less sensitive to changes in income from production. In contrast to the welfare losses of workers, entrepreneur welfare increases, by 0.62% and 0.55%, with each increase in the debt-to-GDP ratio. Measures of consumption equivalence for entrepreneurs reflect improvements to welfare, with entrepreneur consumption needing to be marginally increased in the benchmark steady state for entrepreneurs to be indifferent between the benchmark and the steady states with higher debt. Although entrepreneur welfare gains are relatively larger than worker losses, entrepreneurs make up only 7% of the total population. Decreases in worker welfare thus dominate the increases in entrepreneur welfare.

In an analysis of the distribution of welfare gains across entrepreneurs of different productivity types, the least productive entrepreneurs experience the largest increases in consumption and welfare. Table 11 presents these results in detail. When public debt increases, low shock entrepreneurs increase their steady state consumption, by 2.06% when the debt-to-GDP ratio increases from 80% to 100% and by 3.01% when debt increases again to 120% of GDP. Despite losses in profits, the least productive entrepreneurs see the highest relative increase in total income (see Table 9), allowing them to consume more. They also gain the most with respect to welfare, with welfare for low shock entrepreneurs increasing by 0.71% and 0.64% as debt increases in each steady state. This is reflected as well in the highest measures of consumption equivalence among entrepreneurs. These larger welfare gains result from greater increases in consumption due to significant increases in savings, as shown in panel (b) of Figure 8, and higher capital income.

Table 11: Changes in entrepreneur welfare

	Debt to GDP ratio		
	80%	100%	120%
Low productivity entrepreneurs (60%)			
Consumption variation	0	4.17%	3.86%
Change in consumption	0	2.06%	3.01%
Consumption equivalence ($\times 10^{-3}$)	0	0.145	0.255
Change in welfare	0	0.71%	0.64%
Intermediate productivity entrepreneurs (30%)			
Consumption variation	0	3.28%	3.57%
Change in consumption	0	-0.56%	0.76%
Consumption equivalence ($\times 10^{-3}$)	0	0.044	0.071
Change in welfare	0	0.51%	0.45%
High productivity entrepreneurs (10%)			
Consumption variation	0	5.39%	2.05%
Change in consumption	0	0.89%	-1.88%
Consumption equivalence ($\times 10^{-3}$)	0	0.012	0.018
Change in welfare	0	0.31%	0.27%

Meanwhile, entrepreneurs with higher productivity see smaller relative gains in consumption and welfare. Increasing the steady state level of public debt has divergent effects on consumption, first increasing then decreasing the total consumption of intermediate and high shock entrepreneurs. As shown in Table 9, these types of entrepreneurs see smaller relative increases in income due to lower gains in capital income and a stronger impact of profit losses since income from firm production makes up more of their total income. As a result, their income does not increase by as much as low shock entrepreneurs when the government increases debt, and their consumption does not unambiguously increase. The most productive entrepreneurs experience the smallest welfare gains, with their welfare increasing by 0.31% and then by 0.27% and consumption equivalence, while still positive, measured as the smallest among entrepreneurs. The varying changes in consumption and consumption smoothing for more productive entrepreneurs therefore moderate their gains in welfare, with the overall outcome of higher welfare gains for less productive entrepreneurs. As entrepreneurs with low productivity make up 60% of all entrepreneurs, and entrepreneurs with high productivity make up only 10%, increasing the debt-to-GDP ratio increases total entrepreneur consumption and results in relatively higher welfare gains, as discussed above.

7 Conclusion

Increasing levels of government debt raise questions concerning the benefits and costs to economic efficiency and the effects on already high levels of inequality across wealth and income distributions. In this paper I propose a heterogeneous agent model with incomplete markets and entrepreneurship to analyze these questions. My model captures high levels of inequality by generating differential savings behavior and heterogeneous returns to capital that incorporate scale and type dependence due to endogenizing entrepreneurs' marginal products of capital and a borrowing constraint such that capital investment depends on accumulated wealth. In effect, entrepreneur households are generally wealthier and save more in assets than worker households, resulting in top wealth and income inequality.

In a framework with entrepreneurship and heterogeneous returns to capital, increasing public debt generates additional efficiency and distributional costs. While overall efficiency improves due to the crowding out effect on capital, the reallocation of capital away from entrepreneurs and to the corporate sector limits aggregate productivity. A tighter borrowing constraint on capital investment increases the difficulty that less wealthy entrepreneurs face growing out of financial constraints, while a higher interest rate allows wealthy entrepreneurs to grow faster. More public debt increases savings, with worker savings increasing by relatively more than entrepreneurs, which slightly reduces wealth inequality. Workers, however, are hit harder by losses in labor income and income inequality increases. Total welfare and consumption decrease, but breaking this down across households reveals that entrepreneurs experience welfare gains that are dominated by the welfare losses of workers. In this paper, I restrict my analysis to the comparison of steady state equilibria with different levels of public debt. However, the transition between steady states is likely to impose further costs that weigh on economic efficiency, inequality and welfare. This opens a clear avenue for future research to evaluate the costs and benefits of increasing debt and to propose recommendations on optimal policy.

The model could also be enriched and extended to permit a deeper analysis of the effects of an increase in debt on efficiency and distributions in ways that were beyond the scope of this paper. Adding an overlapping generations component would incorporate additional savings motives of smoothing consumption during retirement and insuring against the risk of a long lifespan. An occupational choice in which households decide to be a worker or an entrepreneur upon the realizations of their labor productivity and entrepreneurial ability shocks would allow the model to better match patterns in household mobility and more closely replicate top shares of wealth and income. Worker labor supply could be endogenized in order to take worker incentives to changes in government policy into account and government taxes could be modified to mirror actual tax systems. Indeed, a more realistic tax system is a key factor in better analyzing the distributional consequences of a rise in public debt. Specifically modeling which taxes finance the debt, and how the tax burden on households depends on scale and type, would allow me to separate out incentive effects from implications for the wealth distribution. I would also be able to distinguish between different kinds of gains and losses across households, such as potentially higher heterogeneity in returns to capital for entrepreneurs, larger differences in savings behavior or changes to worker and entrepreneur consumption levels, in particular when considering heterogeneous marginal propensities to consume.

The framework could be extended to incorporate an illiquid asset in order to better model the illiquidity of investment in private business and capture more nuanced dynamics of wealth accumulation and asset returns. As in Kaplan and Violante (2014), the addition of an illiquid asset increases marginal propensity to consume in the model by generating wealthy hand-to-mouth households. The associated cost of adjusting illiquid assets implies a higher rate of return than on liquid assets. This would amplify heterogeneous returns and introduce a second level of scale dependence, that of the accumulated illiquid assets. As illiquid assets cannot be used to invest in capital, such a framework would reinforce scale dependence with respect to liquid asset holdings in the rate of return to capital. Entrepreneurs would thus have more difficulty to grow out of financial constraints and efficiency in the economy would be further reduced. Higher returns on illiquid assets would also increase top wealth concentration, with the highest returns going to the wealthiest entrepreneur households who hold liquid and illiquid assets and invest in the most capital. Given the impacts on efficiency and the wealth distribution of adding an illiquid asset, increasing public debt in this setup could result in higher efficiency and distributional costs to the economy and more inequality overall.

Furthermore, the borrowing constraint on liquid assets could be loosened to allow house-

holds to borrow. Low asset households, mostly made up of workers, have a high marginal propensity to consume. They will choose to borrow if possible when faced with a low productivity shock, which would reinforce type dependence in savings behavior. The savings of the wealthy, mostly entrepreneurs, would thus finance the borrowing of poor households in addition to financing the public debt, as modeled by Mian et al. (2021). With more poor households in debt and paying the returns on savings of the wealthy, there would be higher baseline wealth inequality. An increase in public debt could increase entrepreneur savings, increasing worker borrowing, imposing a higher burden on workers due to higher costs, and exacerbate wealth and income inequality.

8 References

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9 Appendix

Section A

The Markov process for worker labor productivity is given by

$$\begin{aligned} Z^w &= \{0.646, 0.798, 0.966, 1.169, 1.444\}, \\ \mathbb{P}^w &= \begin{bmatrix} 0.731 & 0.253 & 0.016 & 0.000 & 0.000 \\ 0.192 & 0.555 & 0.236 & 0.017 & 0.000 \\ 0.011 & 0.222 & 0.534 & 0.222 & 0.011 \\ 0.000 & 0.017 & 0.236 & 0.555 & 0.192 \\ 0.000 & 0.000 & 0.016 & 0.253 & 0.731 \end{bmatrix}, \\ \text{invariant distribution} &= [0.165 \quad 0.218 \quad 0.233 \quad 0.218 \quad 0.165]. \end{aligned}$$

The Markov process for entrepreneur firm productivity is given by

$$\begin{aligned} Z^e &= \{0.706, 1.470, 2.234\}, \\ \mathbb{P}^e &= \begin{bmatrix} 0.87 & 0.13 & 0.00 \\ 0.21 & 0.68 & 0.11 \\ 0.16 & 0.17 & 0.67 \end{bmatrix}, \\ \text{invariant distribution} &= [0.603 \quad 0.298 \quad 0.099]. \end{aligned}$$

Section B

The computational algorithm to solve for a benchmark steady state equilibrium of the model is as described below. As the worker and entrepreneur problems are non-linear, an equilibrium is found by iterating on value functions over discrete asset grids. The minimum grid point is set at the borrowing constraint on a . The maximum asset grid point is chosen such that the asset distributions of workers and entrepreneurs in the stationary equilibrium do not have a high mass of households accumulated at the maximum asset level. The intermediate grid points are chosen such that the points are closer together near the minimum asset value where there is high curvature. The points become more spread out as value functions approach linearity.

1. Guess debt level B , interest rate r and tax rate τ , and calculate the wage w implied by the guess on r .
2. Derive decision rules for entrepreneur demands for capital k and labor n , entrepreneur savings a^e and worker savings a^w . N.B. The production output y for each entrepreneur can be found based on the decision rules for capital and labor.
3. Given the decision rules found in step 2, compute the invariant distributions for workers Q^w and entrepreneurs Q^e .
4. Determine corporate inputs:

$$(a) N^c = \int z^w \ell^w dm^w - \int n dm^e,$$

$$(b) K^{c(d)} = \left(\frac{r+\delta}{\alpha A^c L^{e(1-\alpha)}} \right)^{\frac{1}{\alpha-1}}.$$

5. Find taxes and government spending:

- (a) Take as given corporate inputs N^c, K^c calculated in step 4.
- (b) Calculate corporate output $Y^c = A^c(K^c)^\alpha(N^c)^{1-\alpha}$ and total output
 $Y = Y^c + \int y dm^e$.
- (c) Calculate government spending as a fraction of total output $G = 0.15Y$.
- (d) Calculate the tax rate required such that the government budget constraint holds:

$$\tau = \frac{G + rB}{\int (ra^w + wz^w\ell^w) dm^w + \int (ra^e + y - (r + \delta)k - wn) dm^e}.$$

- (e) Guess a new tax rate as a weighted average of the original guess and calculated implied tax rate in 5(d).
- (f) Repeat steps 3-5(e) until tax rates converge.

6. Calculate capital supply:

- (a) Total capital supply: $K^{(s)} = \int a^w dm^w + \int a^e dm^e - B$,
- (b) Capital supplied to corporate sector: $K^{c(s)} = K^{(s)} - \int k dm^e$.

7. Calculate the interest rate required such that corporate capital demand equals corporate capital supply in 6(b):

$$r = \alpha A^c (K^{c(s)})^{\alpha-1} (N^c)^{1-\alpha} - \delta.$$

- 8. Guess a new interest rate as the weighted average of the original guess and the calculated implied interest rate in step 7.
- 9. Repeat steps 2-8 until interest rates converge.
- 10. Calculate the level of debt required such that the public debt to GDP ratio equals the desired percent (e.g. $B = 0.80Y$).
- 11. Guess a new debt level as the weighted average of the original guess and the calculated implied debt level in step 10.
- 12. Repeat steps 2-11 until debt levels converge.

Section C

Consumption equivalence is calculated as the amount by which consumption in the benchmark steady state equilibrium would need to be permanently changed such that households are indifferent between the benchmark steady state and the new steady state with a higher debt-to-GDP ratio. For a given type of household i , take $\Delta^i(a^i, e^i)$ to be this permanent change in consumption to a household's consumption in the benchmark steady state for a household that holds a^i assets in savings and realizes a productivity shock z^i . In the calculations below, I drop the superscript i for simplicity. Let $v^*(a, z)$ be a household's value function in the benchmark steady state and $v^{**}(a, z)$ the value function in the new steady state. Consumption equivalence is calculated such that $v_{\Delta}^*(a, z) = v^{**}(a, z)$.

The benchmark value function can be expressed as

$$v^*(a, z) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \frac{c(a, z)^{1-\mu}}{1-\mu} - \chi \frac{\ell^{1+\varphi}}{1+\varphi} \mid a, z \right]$$

Following Heer and Trede (2003), the value function taking into account the permanent change in consumption can be expressed as

$$v_{\Delta}^*(a, z) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \frac{[(1 + \Delta(a, z))c(a, z)]^{1-\mu}}{1-\mu} - \chi \frac{\ell^{1+\varphi}}{1+\varphi} \mid a, z \right]$$

As I abstract from a labor-leisure choice in my model, since all household inelastically supply a given amount of labor, I drop the labor terms in the above equations. This gives

$$\begin{aligned} v^*(a, z) &= \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \frac{c(a, z)^{1-\mu}}{1-\mu} \mid a, z \right] \\ v_{\Delta}^*(a, z) &= \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \frac{[(1 + \Delta(a, z))c(a, z)]^{1-\mu}}{1-\mu} \mid a, z \right] \end{aligned}$$

which implies that

$$v_{\Delta}^*(a, z) = (1 + \Delta(a, z))^{1-\mu} v^*(a, z).$$

Therefore, setting the value function that incorporates the permanent change in consumption equal to the value function in the new steady state, consumption equivalence for a given household can be derived as

$$\Delta(a, z) = \left(\frac{v^{**}(a, z)}{v^*(a, z)} \right)^{\frac{1}{1-\mu}} - 1$$

Aggregating over households gives

$$\begin{aligned} CEV(a, z) &= \int_{a, z} \Delta(a, z) dm^*(a, z) \\ &= \int_{a, z} \left[\left(\frac{v^{**}(a, z)}{v^*(a, z)} \right)^{\frac{1}{1-\mu}} - 1 \right] dm^*(a, z) \end{aligned}$$

where $m^*(a, z)$ is the time-invariant distribution of households over assets and productivity levels in the benchmark steady state equilibrium.